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SEAT No. :

PA-1004

[Total No. of Pages : 3

[5902]-23
F.Y. B.Sc. (Computer Science)
MATHEMATICS
MTC - 121 : Linear Algebra
(2019 Pattern) (Semester - II) (Paper - I)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of single memory, non-programmable scientific calculator is allowed.*

Q1) Attempt any five of the following. **[10]**

a) Suppose $V = M_{2\times 2}$, a set of matrices of order 2×2 with real entries. we define, $w = \left\{ \begin{bmatrix} a & o \\ c & d \end{bmatrix} \Big/ a, c, d \in \mathbb{R} \right\}$ show that, w is a subspace of v.

b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a non-empty function defined by, $T(x_1, x_2) = (x_1 + x_2 + 1, -4x_1 + x_2, 2x_2)$ Justify, (whether) T is a linear transformation.

c) If $\lambda = -2$, is an eigenvalue of a matrix $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ then find the corrosponding eigenvector.

d) Show that the vector $u = \begin{bmatrix} 12 \\ 3 \\ 5 \end{bmatrix}$ & $v = \begin{bmatrix} 2 \\ -3 \\ -3 \end{bmatrix}$ are orthogonal to each other.

e) Compute the quadratic form of $A = \begin{bmatrix} 3 & -2 \\ -2 & 7 \end{bmatrix}$.

f) Define

   i) Affine combination of vectors.

   ii) Convex combination of vectors.

g) Define 'basis' for vector space.

P.T.O.

Q2) Attempt any three of the following: **[15]**

a) Determine, whether the set of vectors $S = \{(1,0,-2),\ (3,2,-4),\ (-3,-5,1)\}$ forms a basis of $\mathbb{R}^3$.

b) Let $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

Find

i) Eigenvalues of A.

ii) Eigenvector corrosponding ot the largest eigenvalue of A.

c) Let $U = [u_1\ u_2]$, where $u_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} \cdot u_2 = \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$ & $y = \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}$

compute

i) Proj_w^y - where, W = span $\{u_1, u_2\}$

ii) $\{UU^T\}_y$

d) Classify the quadratic form $2x^2 - 4x_1x_2 - x_2^2$ by using the principle axis theorem.

e) Let $B = \{1+t^2,\ t+t^2,\ 1+2t+t^2\}$ be a basis of P_2.

Find the coordinate vector of $p(t) = 1 + 4t + 7t^2$, relative to B.

Q3) Attempt any one of the following. **[10]**

a) Find the bases for the row space, the column space & Null space of A.

Where, $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & 6 & 10 & 7 \end{bmatrix}$.

b) i) Prove that, an indexed set $s=\{\vec{u}_1,...\vec{u}_k\}$ of two or more vectors with $\vec{u}_i, i>1$ is a linear combination of the preceding vectors $\{\vec{u}_1,...,\vec{u}_{i-1}\}$

ii) If $u=\begin{bmatrix}\frac{7}{4}\\ \frac{1}{2}\\ 1\end{bmatrix}$ & $v=\begin{bmatrix}-4\\-1\\8\end{bmatrix}$

then find : 1) A unit vector in the direction of vector u.'

2) $\|u+v\|$.

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