Total No. of Questions : 3]

SEAT No. :

P5131

[Total No. of Pages : 2

[5823] - 203
F.Y.B.Sc.
COMPUTER SCIENCE
Mathematics
MTC-121 : Linear Algebra
(2019 Pattern) (Semester -II)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicates full marks.*
3) *Use of single memory, non-programmable scientific calculator is allowed.*

Q1) Attempt any five of the following. **[10]**

a) Define subspace of a vector space. Give one example of subspace of a vector space $\mathbb{R}^2$.

b) If $\bar{u}=(1,2,-1)$ and $\bar{v}=(2,0,2)$ then find angle between $\bar{u}$ and $\bar{v}$.

c) Write the standard basis for $P_2(\mathbb{R})$. Also write it's dimension.

d) Is the transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by T(x,y) = (2x, 1) is linear? Justify.

e) Define the following terms:

i) Affine set

ii) Convex combination of Vectors

f) Find the matrix of quadratic form given below:

$$Q(x)=3x_1^2+2x_2^2-5x_3^2-6x_1x_2+3x_1x_3-4x_2x_3.$$

g) Find the distance between vectors.

$$X=\begin{bmatrix}10\\-3\end{bmatrix} \text{and } Y=\begin{bmatrix}-1\\-5\end{bmatrix}.$$

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Q2) Attempt any three of the following. **[15]**

a) If W_1 and W_2 are subspaces of a vector space Y, then prove that $W_1 \cap W_2$ is a subspace of V. Is $W_1 \cup W_2$ is a subspace of V? Justify.

b) Find rank of following matrix A and hence write it's nullity.

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 3 & -1 \end{bmatrix}.$$

c) Find all eignvalues of the following matrix A & hence state whether it is diagonalizable.

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}.$$

d) Determine whether $S = \{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ is a basis for $\mathbb{R}^3$ where $\bar{u}_1 = (2,-1,3)$, $\bar{u}_2 = (4,1,3)$ $\bar{u}_3 = (8,-1,8)$.

e) Classify the quadratic form given below $Q(x) = 4x_1^2 - 4x_1x_2 + 4x_2^2$.

Q3) Attempt any one of the following. **[10]**

a) Check whether the following matrix is diagonalizable. If yes find the matrix P that diagonalizes A.

$$A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}.$$

b) i) Express $P = 1 + 2x - x^2$ as a linear combination of $P_1 = 1+x$, $P_2 = 1-x$ and $P_3 = x^2$.

ii) Define orthonormal set. Determine whether the given set $S = \{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ is orthonormal or not, where.

$$\bar{u}_1 = \begin{bmatrix} 1\sqrt{10} \\ 3/\sqrt{20} \\ 3/\sqrt{20} \end{bmatrix} \quad \bar{u}_2 = \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{20} \\ -1/\sqrt{20} \end{bmatrix} \quad \bar{u}_3 = \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

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