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# F.Y. B.Sc. (Computer Science) MATHEMATICS <br> MTC-111 : Matrix Algebra (2019 Pattern) (Semester - I) (Paper-I) 

## Time : 2 Hours]

[Max. Marks : 35
Instructions to the candidates:

1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of single memory, non programmable scientific calculator is allowed.

Q1) Attempt any Five of the following.
a) Let $\mathrm{A}=\left[\begin{array}{ll}4 & -1 \\ 5 & -2\end{array}\right]$. Compute $3 \mathrm{I}_{2}-\mathrm{A}$.
b) Is the matrix $A=\left[\begin{array}{cc}6 & -9 \\ -4 & 6\end{array}\right]$ invertible? Justify.
c) Determine whether the given system is consistent.

$$
\begin{aligned}
& x_{1}+5 x_{2}=7 \\
& -2 x_{1}-7 x_{2}=-5
\end{aligned}
$$

d) What is the condition on matrix A, So that the homogeneous system of linear equations $\mathrm{Ax}=0$ has non-trivial solution?
e) Let $\mathrm{T}: \mathrm{R}^{3} \rightarrow \mathrm{R}^{2}$ be a linear transformation. Find the standard matrix of T , if $\mathrm{T}\left(\bar{e}_{1}\right)=(1,3), \mathrm{T}\left(\bar{e}_{2}\right)=(4,-7)$ and $\mathrm{T}\left(\bar{e}_{3}\right)=(-5,4)$, where $\bar{e}_{1}=(1,0,0)$, $\bar{e}_{2}=(0,1,0)$ and $\bar{e}_{3}=(0,0,1)$.
f) What is the rank of a $4 \times 5$ matrix, whose null space is 3 dimensional?
g) Does the vector $[X]=\left[\begin{array}{l}2 \\ 1\end{array}\right]$ belong to Null A, where $A=\left[\begin{array}{cc}1 & -2 \\ -2 & 4\end{array}\right]$ ?

Q2) Attempt any three of the following.
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a) Find the volume of parallelopiped with one vertex at origin and adjacent vertices are $(1,4,0),(-2,-5,2)$ and $(-1,2,-1)$.
b) Solve the system of linear equations.

$$
\begin{aligned}
& x_{1}-3 x_{2}+4 x_{3}=-4 \\
& 3 x_{1}-7 x_{2}+7 x_{3}=-8 \\
& -4 x_{1}+6 x_{2}-x_{3}=7
\end{aligned}
$$

c) Determine whether $\bar{u}=\left[\begin{array}{c}2 \\ -1 \\ 6\end{array}\right]$ is a linear combination of $\bar{u}_{1}=\left[\begin{array}{c}1 \\ -2 \\ 0\end{array}\right], \bar{u}_{2}=\left[\begin{array}{l}0 \\ 1 \\ 2\end{array}\right]$ and $\bar{u}_{3}=\left[\begin{array}{c}5 \\ -6 \\ 9\end{array}\right]$.
d) Find a basis for null space of A.

Where $A=\left[\begin{array}{ccccc}-3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4\end{array}\right]$
e) Determine whether the vectors $\bar{v}_{1}=\left[\begin{array}{l}0 \\ 0 \\ 2\end{array}\right], \bar{v}_{2}=\left[\begin{array}{c}0 \\ 5 \\ -8\end{array}\right]$ and $\bar{v}_{3}=\left[\begin{array}{c}-3 \\ 4 \\ 1\end{array}\right]$ are linearly independent in $\mathrm{R}^{3}$.

Q3) Attempt any one of the following.
a) Convert the matrix $\mathrm{A}=\left[\begin{array}{ccc}3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0\end{array}\right]$ into $L U$ factorization and use it to

$$
\text { solve } \mathrm{A} x=\mathrm{b} \text {, where } b=\left[\begin{array}{c}
-7 \\
5 \\
2
\end{array}\right]
$$

b) Show that $\mathrm{T}: \mathrm{R}^{2} \rightarrow \mathrm{R}^{2}$ defined by $\mathrm{T}(x, y)=(x+y, x-y)$ is a linear transformation.
c) Prove that the set $\mathrm{S}=\left\{\bar{u}_{1}, \bar{u}_{2}\right\}$ is linearly dependent if and only if one vector is a scalar multiple of the other.

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