Total No. of Questions : 5]

SEAT No. :

P1383

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[5623]-1003
F.Y. B.Sc. (Computer Science)
MATHEMATICS
MTC - 111 : Matrix Algebra
(2019 Pattern) (Paper - I)

Time : 1½ Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *Q.1 is compulsory.*
2) *Solve any three questions from Q.2 to Q.5.*
3) *Figures to the right indicate full marks.*
4) *Use of single memory, non-programmable scientific calculators is allowed.*

Q1) Attempt any five of the following : **[5]**

a) Describe the nature of the solution of the following system of linear equations.

$x + y = 1$

$x - y = 1$

b) Find an elementary matrix E such that EA = I, where $A = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$.

c) If $u = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, then compute

i) $u + 7v$ ii) $\sqrt{2}u$

d) State Rank Nullity theorem for matrix.

e) Suppose a 4 × 7 coefficient matrix for a system of linear equations has 4 pivots. Is the system consistent? How many solutions are there?

f) Write the standard matrix for transformation that gives reflection through x_1 - axis.

P.T.O.

Q2) a) Let $T:\mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Prove that T is one to one if and only if the equation $T(X) = 0$ has only trivial solution. **[6]**

OR

Find the general solution of the following system : **[6]**

$x_1 - 7x_2 + 6x_4 = 5$

$x_3 - 2x_4 = -3$

$-x_1 + 7x_2 - 4x_3 + 2x_4 = 7$

b) Find the determinant of following matrix **[4]**

$$A = \begin{bmatrix} 3 & 4 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 3 \\ 6 & 8 & -4 & -1 \end{bmatrix}.$$

Q3) a) Let $A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$ **[6]**

Use LU decomposition to solve $AX = b$.

OR

If A is a $m \times n$ matrix, $u, v \in \mathbb{R}^n$ and C is a scalar then prove that **[6]**

i) $A(u + v) = Au + Av$.

ii) $A(Cu) = C(Au)$.

b) Determine if the vectors $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$ are linearly dependent. **[4]**

Q4) a) Find inverse of the following matrix A by row reduction method

$$A=\begin{bmatrix}1 & 0 & -2\\ -3 & 1 & 4\\ 2 & -3 & 4\end{bmatrix}.$$ **[6]**

OR

Find the basis for ColA and for NulA, where $A=\begin{bmatrix}4 & 5 & 9 & -2\\ 6 & 5 & 1 & 12\\ 3 & 4 & 8 & -3\end{bmatrix}$. **[6]**

b) Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be a linear transformation, defined as $T(x_1,x_2)=(x_1-2x_2,-x_1+3x_2,3x_1-2x_2)$ Find X such that T(X) = (–1, 4, 9). **[4]**

Q5) Attempt any two of the following : **[10]**

a) Find AB using the partitioned matrices shown below,

$$A=\left[\begin{array}{ccc|cc}2 & -3 & 1 & 0 & 4\\ 1 & 5 & -2 & 3 & -1\\ \hline 0 & -4 & -2 & 7 & 1\end{array}\right] \text{ and } B=\left[\begin{array}{cc}6 & 4\\ -2 & 1\\ -3 & 7\\ \hline -1 & 3\\ -5 & 2\end{array}\right]$$

b) Let $T:\mathbb{R}^2\to\mathbb{R}^2$ be a linear transformation such that,

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}2\\5\end{bmatrix}, T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}-1\\6\end{bmatrix}$$

Find the images of $\begin{bmatrix}x_1\\x_2\end{bmatrix}$ and $\begin{bmatrix}-5\\-3\end{bmatrix}$.

c) Use Cramer's rule to compute the solutions of following system :

$x+y+2z=7$

$-x-2y+3z=6$

$3x-7y+6z=1$.

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