Total No. of Questions : 9]

SEAT No. :

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[Total No. of Pages : 5

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S.E. (Production Engg. & Industrial Engineering) (Production S.W/Robotics & Automation (RA))

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207007)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.No. 1 is compulsory.*
2) *Attempt Q.No.2 or Q.No.3, Q.No.4 or Q.No.5, Q.No.6 or Q.No.7, Q.No.8 or Q.No.9.*
3) *Figures to the right indicate full marks.*
4) *Use of logarithmic table, slide rule, electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) Choose the correct option of the following.

a) $\nabla\phi$ for $\phi = x^2 + y^2 + z^2$ at (1, 1, 1) is ______ **[2]**

i) $\bar{i}+\bar{j}+\bar{k}$ ii) $2\bar{i}+2\bar{j}+2\bar{k}$

iii) $4\bar{i}+4\bar{j}+4\bar{k}$ iv) None of these

b) The most general solution of heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ if (A) u is finite for all t, (B) $u(0, t) = 0$ and $u(l, t) = 0$ for all t, (C) $u(x, 0) = f(x)$ for $0 \le x \le l$ is $u(x, t) =$ ______ **[2]**

i) $(C_4\cos mx + C_5\sin mx)e^{-m^2t}$

ii) $C_1e^{mx} + C_2e^{-mx}$

iii) $C_1\cos mx + C_2\sin mx$

iv) $(C_1\cos mx + C_2\sin mx)(C_3\cos cmtx + C_4\sin cmt)$

c) If $n = 100$ and $P = 0.01$ then by Poisson distribution, $P(r = 0) =$ ______ **[2]**

i) $\frac{1}{e}$ ii) $\frac{2}{e}$

iii) $\frac{3}{e}$ iv) $\frac{4}{e}$

P.T.O.

d) If the first four central moments of a distribution are 0, 2.5, 0.7 and 18.75 then the coefficient of Kurtosis β_2 is ________. **[2]**

i) 0 ii) 1

iii) 2 iv) 3

e) The regression line of x on y is given by __________. **[1]**

i) $(x-\bar{x}) = b_{xy}(y-\bar{y})$ ii) $(y-\bar{y}) = b_{yx}(x-\bar{x})$

iii) $(x-\bar{x}) = b_{yx}(y-\bar{y})$ iv) $(y-\bar{y}) = b_{xy}(x-\bar{x})$

f) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then $\nabla \cdot \bar{r} =$ _______ **[1]**

i) 1 ii) 2

iii) 3 iv) 0

Q2) a) Fit a straight line of the form $y = ax + b$ to the following data, **[5]**

x	100	120	140	160	180	200
y	0.9	1.1	1.2	1.4	1.6	1.7

b) The first four moments of a distribution about 4 are –1.4, 17, –30 and 108. Obtain the first four central moments and coefficient of Skewness and Kurtosis. **[5]**

c) Calculate the coefficient of correlation from the following information, $n = 10$, $\Sigma x = 40$, $\Sigma x^2 = 190$, $\Sigma y^2 = 200$, $\Sigma xy = 150$, $\Sigma y = 40$ **[5]**

OR

Q3) a) Fit a straight line of the form $y = ax + b$ to the following data, **[5]**

x	1	3	4	5	6	8
y	–3	1	3	5	7	11

b) The first four moments of a distribution about the values 5 are 2, 20, 40 & 50. Obtain the first four central moments, mean and coefficient of Skewness and Kurtosis. **[5]**

c) If the two lines of regression are $9x + y - \lambda = 0$ and $4x + y = \mu$ and the mean of x and y are 2 and –3 respectively. Find values of λ, μ and correlation coefficient between x and y. **[5]**

Q4) a) A throw is made with two dice. Find the probability that **[5]**

(i) the sum is 7 or less (ii) the sum is a perfect square

b) The mean and variance of a binomial distribution are 6 and 2 respectively. Find $P(r \geq 1)$. **[5]**

c) A life-time of a certain component has a normal distribution with mean of 400 hours and standard deviation of 50 hours. Assuming a normal sample of 1000 components, find the number of components whose life - time lies between 340 to 465 hours.

Whose life-time lies between 340 to 465 hours. **[5]**

[Given : A(Z = 1.2) = 0.3849, A(Z = 1.3) = 0.4032]

OR

Q5) a) On an average a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have three or less defectives? **[5]**

b) Number of road accidents on a highway during a month follows a Poisson distribution with mean 5. Find the probability that in a certain month number of accidents on the highway will be **[5]**

i) less than 3

ii) more than 3

c) Demand for a particular spare part in a factory was found to vary from day to day. In a sample study the following information was obtained. **[5]**

Days	Mon	Tue	Wed	Thur	Fri	Sat
No. of Parts demanded	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week.

$\left[\text{Given} : \chi^2_{5,0.05} = 11.07\right]$

Q6) a) Find the direction derivative of the function $\phi = e^{2x-y-z}$ at (1, 1, 1) in the direction of the tangent to the curve $x = e^{-t}$, $y = 2\sin t + 1$ $z = t - \cos t$ at $t = 0$. **[5]**

b) Show that the vector field $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational find the scalar potential φ such that $\bar{F} = \nabla\phi$ **[5]**

c) Evaluate $\int \bar{F}\cdot d\bar{r}$ for $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the curve define by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. **[5]**

OR

Q7) a) Find the direction derivative of $\phi = xy^2 + 2xy + zx$ at the point (2, –1, 1) in the direction $2\bar{i} + \bar{j} + 3\bar{k}$ **[5]**

b) Show that (any one) **[5]**

i) $\nabla^2\left[r^n \log r\right] = \left[n(n+1)\log r + 2n + 1\right]r^{n-2}$

ii) $\nabla \cdot \left[r\nabla\left(\frac{1}{r^n}\right)\right] = \frac{n(n-2)}{r^{n+1}}$

c) Apply Greens Theorem to evaluate $\int_C \sin y dx + x(1+\cos y)dy$ where C is the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1, z = 0$. **[5]**

Q8) a) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions : **[8]**

i) $u(0, t) = 0°C$

ii) $u(1, t) = 0°C$

iii) $u(x, 0) = 50°C$ for $0 \leq x \leq 1$, where 1 is the length of the bar

b) A string is stretched and fastened to two points 5 units apart. Motion is started by displacing the string in the form $u = a\sin\frac{\pi x}{5}$ from which it is released at time t = 0. Find the displacement $u(x, t)$ from one end using wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ **[7]**

OR

Q9) a) Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions : **[8]**

i) $u(0, y) = 0$

ii) $u(\pi, y) = 0$

iii) $u(x, \infty) = 0$ for $0 < x < \pi$

iv) $u(x, 0) = u_0$ for $0 < x < \pi$

b) Using Fourier sine transform, solve the equation

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty,\ t > 0$$

Subject to the following conditions

i) $u(0, t) = 0 \quad t > 0$

ii) $u(x,0) = e^{-x} \quad x > 0$

iii) u and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$ **[7]**