Total No. of Questions : 9]

SEAT No. :

PD4130

[Total No. of Pages : 4

[6402]-91

S.E. (Production and Industrial Engineering/(Production S.W/ Robotics & Automation)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207007)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory. Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Figures to the right indicate full marks.*
3) *Assume suitable data, if necessary.*
4) *Use of electronic pocket calculator is allowed.*

Q1) a) The first and second moments of the distribution about the value 3 are 2 and 20. second moment about the mean is **[2]**

i) 12

ii) 14

iii) 16

iv) 20

b) The mean and variance of binomial probability distribution are $\frac{5}{4}$ and $\frac{15}{16}$ respectively. Probability of failure q in a single trial is equal to **[2]**

i) $\frac{1}{2}$

ii) $\frac{15}{16}$

iii) $\frac{1}{4}$

iv) $\frac{3}{4}$

c) $\nabla(\overline{a}\cdot\overline{r})$ where $\overline{a} = a_1\overline{i} + a_2\overline{j} + a_3\overline{k}$, $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ is equal to **[2]**

i) 0

ii) $\overline{a}$

iii) $\overline{r}$

iv) 1

d) $\nabla \times \bar{r}$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ is equal to **[1]**

i) 0

ii) $\frac{1}{r}\bar{r}$

iii) $\bar{r}$

iv) 3

e) For the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with general solution $u(x, t) = (C_4 \cos mx + C_5 \sin mx)\, e^{-m^2 t}$ if $u(0, t) = 0, \ \forall t$ then **[2]**

i) $C_5 = 0$

ii) $C_4 = 0$

iii) $m = 0$

iv) None of these

f) Mean of binomial probability distribution is **[1]**

i) nq

ii) n^2p

iii) npq

iv) np

Q2) a) Fit a straight line of the form $y = ax + b$ to the following data **[5]**

x	6	2	10	4	8
y	9	11	5	8	7

b) Calculate the first four moments about the mean of the following distribution. **[5]**

x	2	2.5	3	3.5	4	4.5	5
y	4	36	60	90	70	40	10

c) Find the coefficient of correlation for the following table **[5]**

x	35	34	40	43	56	20	38
y	32	30	31	32	53	20	33

OR

Q3) a) Fit a straight line to the following data. **[5]**

x	0	5	10	15	20	25
y	12	15	17	22	24	30

b) First four moments of distribution about the value 5 are 2, 20, 40 and 50. Find first four central moment also comment on skewness and kurtosis. **[5]**

c) Obtain the regression lines for the following table **[5]**

x	10	14	19	26	30	34	39
y	12	16	18	26	29	35	38

Q4) a) On an average a box containing 10 articles is likely to have 2 defective. If we consider a consignment of 100 boxes. How many of them are expected to have three or less defectives. **[5]**

b) A can hit the target 1 out of 4 times. B can hit the target 2 out of 3 times, C can hit the target 3 out of 4 times. Find the probability of at least two hit the target. **[5]**

c) The mean weight of 500 students is 63 kgs and the standard deviation is 8 kgs. Assuming that the weights are normally distributed. Find how many students weight 52 kgs? The weights are recorded to the nearest kg **[5]**

(Given: Area corresponding to 1.44 is 0.4251

Area corresponding to 1.31 is 0.4049)

OR

Q5) a) Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that they are both king if the first card drawn is not replaced. **[5]**

b) In a poisson distribution if $p\ (r = 1) = 2\ p\ (r = 2)$ find $p\ (r = 3)$ **[5]**

c) A nationalized bank utilizes 4 teller windows to render fast service to customers on a particular day 800 customers were observed they were given service at the different windows as follows: **[5]**

No. of windows	1	2	3	4
Expected no.of customer	150	250	170	230

Test whether customers are uniformly distributed

(Given: χ_3^2 ; 0.05 = 7.815)

Q6) a) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ towards the point $(2, 1, -1)$ **[5]**

b) Show that $\overline{F} = (6xy + z^3)\overline{i} + (3x^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$ is irrotational. Find scalar ϕ such that $\overline{F} = \nabla\phi$. **[5]**

c) Evaluate $\int_c \overline{F}.d\overline{r}$ where $\overline{F} = (2xy + 3z^2)\overline{i} + (x^2 + 4yz)\overline{j} + (2y^2 + 6xz)\overline{k}$ and C is the straight line joining (0, 0, 0) and (1, 1, 1) **[5]**

OR

Q7) a) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point (1, 2, 3) in the direction of $4\bar{i} - 2\bar{j} + \bar{k}$. **[5]**

b) Show that (any one) **[5]**

i) $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^2}\right) = \frac{\bar{a}}{r^2} - 2\frac{(\bar{a}\cdot\bar{r})\bar{r}}{r^4}$

ii) $\nabla\cdot\left[r\nabla\frac{1}{r}\right] = \frac{-1}{r^2}$

c) Evaluate $\int_c \bar{F}.d\bar{r}$ for $\bar{F} = (2x+y)\bar{i} + (3y-x)\bar{j}$ and C is parabolic curve $y^2 = x$ joining points (0, 0) and (1, 1) **[5]**

Q8) a) If $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length l, fixed at both ends, find solution with conditions: **[8]**

i) $y(0, t) = 0$

ii) $y(l, t) = 0$

iii) $\frac{\partial y}{\partial t} = 0$ at $t = 0$

iv) $y(x, 0) = k(lx - x^2),\ 0 < x < l$

b) Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ if **[7]**

i) $u(0, t) = 0$

ii) $u_x(l, t) = 0$

iii) $u(x, t)$ is bounded

iv) $u(x,0) = \frac{u_0 x}{l}$

OR

Q9) a) An infinitely long uniform uniform metal plate is enclosed between lines $y = 0$ and $y = 2$ meters for $x > 0$. The temperature is zero along the edges $y = 0, y = 2$ meters and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature distribution $f(x, y)$. **[8]**

b) Using Fourier transform. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < \infty,\ t > 0$ **[7]**

Subject to conditions

i) $u(0, t) = 0,\ t > 0$

ii) $u(x,0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

iii) $u(x, t)$ is bounded

ⓘⓘⓘⓘ