Total No. of Questions : 9]

SEAT No. :

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# S.E. (Production Engineering & Industrial Engg.) (Robotics & Automoble) (Sandwich)

## ENGINEERING MATHEMATICS-III

## (2019 Pattern) (Semester-III) (207007)

*Time : 2½ Hours]* *[Max. Marks : 70*

***Instructions to the candidates:***

1) ***Question No.1 is compulsory.***
2) ***Attempt Q2 or Q3,Q4 or Q5,Q6 or Q7,Q8 or Q9***
3) ***Figures to the right indicates full marks.***
4) ***Assume suitable data, if necessary.***
5) ***Use of electronic pocket calculator is allowed.***
6) ***Neat diagrams must be drawn wherever necessary.***

***Q1)*** choose correct option:

a) From the given information S.D. of $x = 4$, S.D. of $y = 1.8$ and coefficient of regression of $y$ on $x$ is 0.32. The coefficient of correlation is **[2]**

i) 0.711 ii) 0.622

iii) 0.743 iv) 0.543

b) The coefficient of Kurtosis $\beta_2$ is given by **[1]**

i) $\frac{\mu_3^2}{\mu_2^3}$ ii) $\frac{\mu_4^2}{\mu_2^4}$

iii) $\frac{\mu_4}{\mu_2^2}$ iv) $\frac{\mu_2^3}{\mu_3^2}$

c) If A and B are any two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$ then P(A/B) is **[2]**

i) $\frac{1}{3}$ ii) $\frac{3}{4}$

iii) $\frac{1}{4}$ iv) $\frac{2}{3}$

d) Two dice are thrown, the probability of getting a score 10 is **[1]**

i) $\frac{1}{12}$ ii) $\frac{1}{6}$

iii) $\frac{1}{5}$ iv) $\frac{2}{3}$

***P.T.O.***

e) Value of $\nabla^2\left(\frac{1}{r}\right)$ is **[2]**

i) $\frac{-\bar{r}}{r^3}$ ii) $\frac{\bar{r}}{r^2}$

iii) $\frac{1}{r^4}$ iv) 0

f) For the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with general solution **[2]**

$u(x, t) = (C_4 \cos mx + C_5 \sin mx).\ e^{-m^2 t}$ if $u(0, t) = 0,\ \forall t$ then

i) $C_5 = 0$ ii) $C_4 = 0$

iii) $m = 0$ iv) None of these

***Q2)*** a) Fit a straight line for the following data of the form $y = ax + b$ **[5]**

| $x$ | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|
| $y$ | 1 | 2 | 3 | 4 | 5 |

b) The first four moments about the mean 3.5 are 0.058064, 0.451612, 0.082259 and 0.5. Calculate the first four central moments. **[5]**

c) Compute correlation coefficient for the following data. **[5]**

| $x$ | 152 | 158 | 169 | 182 | 160 | 166 | 182 |
|---|---|---|---|---|---|---|---|
| $y$ | 198 | 178 | 167 | 152 | 180 | 170 | 162 |

OR

***Q3)*** a) For the tabulated values of $x$ and $y$ given below, fit a straight line of the form $y = mx + c$ **[5]**

| $x$ | 1.0 | 3.0 | 5.0 | 7.0 | 9.0 |
|---|---|---|---|---|---|
| $y$ | 1.5 | 2.8 | 4.0 | 4.7 | 6.0 |

b) If $\sum f = 27$, $\sum fx = 91$, $\sum fx^2 = 359$, $\sum fx^3 = 1567$, $\sum fx^4 = 7343$. Find the first four moments about origin. **[5]**

c) Find the regression line of $y$ on $x$ from the data below. **[5]**

| $x$ | 65 | 63 | 67 | 64 | 68 | 62 | 70 | 66 | 68 | 67 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 68 | 66 | 68 | 65 | 69 | 66 | 68 | 65 | 71 | 67 |

***Q4)*** a) 20% of bolts produced by a machine are defective. Determine the probability that out of 4 bolts chosen at random **[5]**

i) 1 bolt is defective

ii) almost 2 bolts are defective

b) Number of road accidents on a highway during a month follows poisson's distribution with mean 5. Find the probability that in a certain month number of accidents on the highway will be **[5]**

i) less than 3

ii) more than 3

c) Suppose heights of students follows normal distribution with mean 190cm and variance 80 $cm^2$. In a school of 1000 students how many would you expect to be above 200cm tall. (Given Area A = 0.3686) **[5]**

OR

***Q5)*** a) A die is thrown twice X denote the sum of digits in two throws. Find the mathematical expectation of X. **[5]**

b) The average number of misprints per page of a book is 1.5. Assuming the distribution of number of misprints to be poisson, find. **[5]**

i) The probability that a particular book is free from misprint.

ii) Number of pages containing more than one misprint if the book contains 900 pages.

c) A coin is so biased that appearance of head is twice likely as that of tail. If a throw is made 6 times, find the probability that atleast 2 heads will appear. **[5]**

***Q6)*** a) Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = x^2\bar{i} + xy\bar{j}$ and C is the are of the parabola $y = x^2$ joining (0, 0) and (1, 1) **[5]**

b) Show that the vector field given by **[5]**

$\bar{F} = (y^2 \cos x + z^2)\,\bar{i} + (2y \sin x)\,\bar{j} + 2xz\bar{k}$ is irrotational-

Find scalar potential $\phi$ such that $\bar{F} = \nabla\phi$

c) Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at (1, –1, 1) along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$ **[5]**

OR

**Q7)** a) Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (xy + y^2)\,\bar{i} + x^2\,\bar{j}$ and C is the straight line joining (0, 0) to (2, 2), the equation of the straight line is $y = x$. **[5]**

b) Prove that (any one) **[5]**

i) $\nabla\left(\frac{\bar{a}.\bar{r}}{r^3}\right) = \frac{\bar{a}}{r^3} - \frac{3(\bar{a}.\bar{r})\bar{r}}{r^5}$

ii) $\nabla^2\left(\frac{1}{r^2}\right) = \frac{2}{r^4}$

c) Find the directional derivative of $\phi = 2x + y + z^2$ at (2, −1, 1) along the vector $2\bar{i} + \bar{j} + 2\bar{k}$. **[5]**

**Q8)** a) If $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length $l$ fixed at both ends, find the solution with boundary conditions. **[8]**

i) $y(0, t) = 0, \forall t$

ii) $y(l, t) = 0, \forall t$

and initial conditions

iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

iv) $y(x, 0) = a \sin\left(\frac{\pi x}{l}\right)$

b) Solve $\left(\frac{\partial v}{\partial t}\right) = K\frac{\partial^2 v}{\partial x^2}$ if **[7]**

i) $V \neq \infty$ as $t \to \infty$

ii) $\left(\frac{\partial v}{\partial x}\right)_{x=0} = 0, \forall t$

iii) $V(l,t) = 0, \forall t$

iv) $V(x,0) = v_0$, for $0 < x < l$.

OR

***Q9)*** a) If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length $l$ fixed at both ends, find the solution with boundary conditions, **[8]**

i) $y(0, t) = 0$

ii) $y(l, t) = 0$

and initial conditions

iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

iv) $y(x, 0) = k(lx - x^2),\ 0 \le x \le l$

b) Solve the equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ with conditions **[7]**

i) V = 0 when $y \to +\infty$ for all x.

ii) V = 0 when $x = 0$ for all values of y.

iii) V = 0 when $x = 1$ for all values of y.

iv) $V = x(1 - x)$ when $y = 0$ for $0 < x < 1$.