Total No. of Questions : 9]

SEAT No. :

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S.E. (Production and Industrial Engineering)/(Robotics & Automation Engg.)/(Sandwich)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207007)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Q.1 is compulsory. Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Figures to the right indicate full marks.*
3) *Assume suitable data, wherever necessary.*
4) *Use of electronic pocket calculator is allowed.*

Q1) a) The first four moments about the working mean 5 are –1, 10, 11, 16 then value of second moment μ_2 about mean is given by **[2]**

i) 10 ii) 9

iii) 8 iv) 7

b) Mean and standard deviation of a Binomial distribution are 25 and 5 respectively. Number of trials 'n' is **[2]**

i) 42 ii) 40

iii) 9 iv) 44

c) $\nabla^2\left(\frac{1}{r^2}\right)$ **[2]**

i) $\frac{1}{r^3}$ ii) $\frac{2}{r^4}$

iii) $\frac{-2}{r^4}\bar{r}$ iv) $\frac{6}{r^4}$

d) For a constant vector $\bar{a}, \nabla(\bar{a}.\bar{r}) =$ **[1]**

i) $\bar{a}$ ii) $3\bar{a}$

iii) $\bar{r}$ iv) 0

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e) Most general solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ representing metal plate having length x & breadth $y \to \infty$ is **[2]**

i) $(C_1 \cos mx + C_2 \sin mx)(C_3 e^{my} + C_4 e^{-my})$

ii) $(C_1 e^{mx} + C_2 e^{-mx})$

iii) $(C_1 e^{mx} + C_2 e^{-mx})$

iv) $(C_1 \cos hmx + C_2 \sin hmx)(C_3 \cos my + C_4 \sin my)$

f) If probability of success $p = 0.7$ then probability of failure $q =$ **[1]**

i) 0.7 ii) 1.7

iii) –0.7 iv) 0.3

Q2) a) Fit a Straight line of the from $y = ax + b$ to the following data by least square method **[5]**

x	0	5	10	15	20	25
y	12	15	17	22	24	30

b) Calculate the first four moments about the mean of the following distribution **[5]**

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

c) Obtain the correlation coefficient between population density (per square miles) & depth rate (per thousand persons) from data ralated to 5 cities. **[5]**

x	200	500	400	700	300
f	12	18	16	21	10

OR

Q3) a) Fit a Straight line of the from $y = ax + b$ following data, by using method of least squares **[5]**

x	0	1	2	3	4	5	6	7
y	–5	–3	–1	1	3	5	7	9

b) The first four moments of a distribution about the value 2 are –2, 12, –20 & 100 calculate **[5]**

i) First four central moments.

ii) Coefficients of Skewness & Kurtosis

c) Find the regression line of y on x for the following data **[5]**

x	10	14	18	22	26	30
y	18	12	24	6	30	36

Q4) a) A set of five similar coins is tossed 210 times & the result is **[5]**

No. of Heads	0	1	2	3	4	5
Frequency	2	5	20	60	100	23

Test the hypothesis that the data follow a binomial distribution. $\left[\text{Given } \chi^2_{5:0.05} = 5.991\right]$

b) A manufacturer of cotter pins known that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins & guarantees that not more than 5 pins will be defective in a box find the approximate probability that a box will fail to meet the guaranteed quality. **[5]**

c) A random sample of 200 screws is drawn from a population with which represent size of screws. If a sample is distribution normally with a mean 3.15 cm & standard 0.025cm find the expected number of screws whose size fall between 3.12 & 3.2 cm. [Given A(1.2) = 0.3849 ; A(2) = 0.4772] **[5]**

OR

Q5) a) On an average box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes how many of them are expected to have three or less defectives? **[5]**

b) Number of road accident on a highway during a month follows a Poisson distribution with mean 5. Find the probability that in a certain month number of accidents on the highway will be. **[5]**

i) less than 3

ii) between 3 and 5

c) In experiment on pea breeding, the following frequencies of seeds were obtained **[5]**

Round & green	Wrinkled green	Round & yellow	Wrinkled & yellow	Total
222	120	32	150	524

Theory predicts that the frequencies should be in proportion 8 : 2 : 2 : 1 Examine the correspondence between theory & experiment.

$\left[\text{Given } \chi^2_{3:0.05} = 7.815\right]$

Q6) a) Find the directional derivative of $\phi = xy + yz^2$ at (1, –1, 1) along the line $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-1}{2}$. **[5]**

b) Show that vector field $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - xz)\overline{j} + (z^2 - xy)\overline{k}$ is irrotational. Find scalar ϕ such that $\overline{F} = \nabla\phi$. **[5]**

c) Evaluate $\int_C \overline{F}.d\overline{r}$ along straight line joining (0, 0) and (1, 1) where $\overline{F} = (2x + y^2)\overline{i} + (3y - 4x)\overline{j}$. **[5]**

OR

Q7) a) Find the directional derivative of $\phi = x + y^2 + z^3$ at the point (1, 1, –1) along the direction of $2\overline{i} - \overline{j} + 3\overline{k}$. **[5]**

b) Show that (any one) : **[5]**

i) $\nabla\left(\frac{\overline{a}.\overline{r}}{r}\right) = \frac{\overline{a}}{r} - \frac{(\overline{a}.\overline{r})\overline{r}}{r^3}$

ii) $\nabla^2\left(\nabla.\left(\frac{\overline{r}}{r^2}\right)\right) = \frac{2}{r^4}$

c) Evaluate $\oint_C \overline{F}.d\overline{r}$ for a closed curve which is given by $x^2 + y^2 = 1, z = 0$ where $\overline{F} = \cos y\,\overline{i} + x(1 - \sin y)\overline{j}$ by using Green's Lemma. **[5]**

Q8) a) A string is stretched and fastened to two points distanced one meter apart is displaced in to the form $y(x, 0) = x$ from which it is released at $t = 0$. Determine the displacement of the string at a distance x from one end. **[8]**

b) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, if **[7]**

i) $u(0, t) = 0°C$

ii) $u(1, t) = 0°C$

iii) $u(x, 0) = 50°C, 0 < x < 1$

iv) $u(x, t)$ is finite, $\forall t$

OR

Q9) a) Using Fourier transform, solve the equation

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t > 0$ subject to the conditions. **[8]**

i) $u(0, t) = 0, t > 0$

ii) $u(x, 0) = e^{-x}, x > 0$

iii) $u \to 0$ and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$

b) Determine the distribution of temperature in the semi infinite medium $x \geq 0, 0 < y < 1$ when the end $x = 0$ is maintained at zero temperature and the initial distribution of temperature is $f(x)$. **[7]**

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