Total No. of Questions : 9]

SEAT No. :

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S.E. (P.E. & I.E./Production SW/RA)

ENGINEERING MATHEMATICS-III

(2019 Pattern) (Semester-III) (207007)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory. Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Figures to the right indicate full marks.*
3) *Use graph paper for Graphical Solution.*
4) *Use of electronic pocket calculator is allowed.*
5) *Assume suitable data if necessary.*

Q1) a) μ_2=16, μ_4=162. coefficient of kurtosis β_2 is given by. **[2]**

i) 1 ii) 1.51

iii) 0.63 iv) 1.69

b) A box contains 100 bulbs out of which 10 are defective. A sample of 5 bulbs are drawn. The probability that none is defective is **[2]**

i) $\left(\frac{1}{10}\right)^5$ ii) $\left(\frac{1}{2}\right)^5$

iii) $\left(\frac{9}{10}\right)^5$ iv) $\frac{9}{10}$

c) $\nabla.\bar{r}$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ is equal to **[2]**

i) 0 ii) 1

iii) $x^2 + y^2 + z^2$ iv) 3

d) A vector field $\bar{F}$ is irrotational if **[1]**

i) $\nabla.\bar{F} = 0$ ii) $\nabla\times\bar{F} = 0$

iii) $\bar{F}.\bar{F} = 0$ iv) $\bar{F}\times\bar{F} = 0$

P.T.O.

e) If $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (Represents the vibrations of a string of length l fixed at both ends) with general solution $y(x,t) = (C_1 \cos mx + C_2 \sin mx)(C_3 \cos cmt + C_4 \sin cmt)$ then $Y(0,t)=0$ implies. **[2]**

i) $C_2=0$ ii) $C_4=0$

iii) $C_3=0$ iv) $C_1=0$

f) Variance of binomial probability distribution is **[1]**

i) npq ii) np

iii) np^2q iv) npq^2

Q2) a) Fit a straight line of the form $y = ax + b$ to the following data. **[5]**

x	1	3	5	7	9
y	1.5	2.8	4.0	4.7	6

b) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about mean also evaluate β_1 and β_2 **[5]**

c) Calculate coefficient of correlation from the following information.

$n = 10$ $\sum x = 40$, $\sum x^2 = 190$, $\sum y^2 = 200$, $\sum xy = 150$ $\sum y = 40$ **[5]**

OR

Q3) a) Fit a straight line of the form $y = ax + b$ to the following data. **[5]**

x	1	3	4	5	6	8
y	–3	1	3	5	7	11

b) Calculate the first four central moments for the following frequencies. **[5]**

x	0-10	10-20	20-30	30-40	40-50
y	6	26	47	15	6

c) The regression equations are $8x - 10y + 66 = 0$ and $40x - 18y = 214$. The value of variance of X is 9. find **[5]**

i) Mean values of x & y

ii) Correlation coefficient between x & y

OR

Q4) a) If mean and variance of a binomial distribution are 12 and 3 respectively, find $P(r \geq 1)$ **[5]**

b) An unbaised coin is thrown 10 times. Find the probability of getting exactly 6 heads and atleast 6 heads using binomial distribution. **[5]**

c) An aptitude test for selecting officers in a bank conducted on 1000 candidates. The average score is 42 and standard deviation of score is 24. Assuming normal distribution for the score find; **[5]**

i) The number of candidates exceed 60

ii) The number of candidates score lies between 30 and 60

[Given A (0.75)=0.2734, A (0.5)=0.1915]

OR

Q5) a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001 determine the probability that out of 2000 individuals more than 2 individuals suffer a bad reaction. **[5]**

b) On an average 20% of the workers in an industry suffer with a certain diseases. If 12 workers are chosen from the industry, find the probability that exactly 2workers suffer from the disease. **[5]**

c) A die is tossed 60 times and frequency of each face in indicated below

x	1	2	3	4	5	6
y	5	7	5	14	13	6

Test the die is fair $\left[\text{Given } \chi^2_{0;0.05} = 11.07\right]$ **[5]**

OR

Q6) a) Find the directional derivative of $\phi = x^2yz^3$ at the point (2,1,–1) in the direction of $3\bar{i} + 4\bar{k}$ **[5]**

b) Show that $\bar{F} = (y^2 \cos x + z^2)\bar{i} + (2y \sin x)\bar{j} + 2xz\bar{k}$ is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$ **[5]**

c) Evaluate $\int_c \bar{F}.d\bar{r}$ along the straight line joining (0,0) and (3,2) where

$\bar{F} = (2x + y)\bar{i} + (3y - x)\bar{j}$ **[5]**

OR

Q7) a) Find the directional derivative of $\phi = 2x - y^3 + z^2$ at the point (2,1,1) in the direction of $\bar{i} + \bar{j} + \bar{k}$ **[5]**

b) Show that. (any one) **[5]**

i) $\nabla.\left[r\nabla\frac{1}{r^n}\right] = \frac{n(n-2)}{r^{n+1}}$

ii) $\nabla^2\left(\frac{\bar{a}.\bar{b}}{r}\right) = 0$

c) Using Green's theorem evaluate $\oint_C \bar{F}.d\bar{r}$ where $\bar{F} = (2x+y)\bar{i} + (5x-y)\bar{j}$ and C is circle $x^2 + y^2 = 16$, $z = 0$ **[5]**

Q8) a) If $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length l, fixed at both ends, find solution with the conditions. **[8]**

i) $u\ (0,t)=0$

ii) $u\ (l,t)=0$

iii) $\frac{\partial u}{\partial t} = 0$ at $t = 0$

iv) $u(x,0) = a\sin\left(\frac{\pi x}{l}\right)$

b) Solve $\frac{\partial v}{\partial -1} = K\frac{\partial^2 v}{\partial x^2}$ if **[7]**

i) $v\ (0,t) = 0$

ii) $v(1,t) = 0$

iii) $v_x = 0$ at $x = 0$

iv) $v(x_1,0)=v_0$ for $0 < x < 1$

OR

Q9) a) An infinitely long plane uniform plate is bounded by two parallel edges in the y direction and an end at right angles to them. The breadth of the plate is π. This end is maintained at temperature u_0 at all points and other edges at zero temperature. Find the steady state temperature function $u(x,y)$ **[8]**

b) Use fourier transform to solve. **[7]**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, t > 0$$

Subject to conditions

i) $\left(\frac{\partial u}{\partial t}\right)_{x=0} = 0, t > 0$

ii) $u(x,0) = \begin{cases} x & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

iii) $u,(x,t)$ is bounded

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