Total No. of Questions : 9]

SEAT No. :

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S.E. (Production & Industrial Engineering)(Production -S.W/RA)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207007)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No.s 1 are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of logarithmic tables slide rule, electronic pocket calculator is allowed.*
4) *Assume suitable data, if necessary.*

Q1) Choose the correct option:

a) First four moments of a distribution about the value 4 of the variable are –1.5, 17, –30 and 108. The second moment about mean is **[2]**

i) 14.75 ii) 15.75

iii) 23.75 iv) 12.75

b) If a random variable X follows binomial distribution with n no. of trials p as probability of success & r as no. of success then $p(X=r)$ is **[2]**

i) ${}^nC_r p^r q^{n-r}$ ii) ${}^nC_r p^r$

iii) ${}^nC_r q^r$ iv) ${}^nC_r p^r q$

c) Two events A and B are mutually exclusive $P(A)=\frac{1}{5}$ $P(B)=\frac{1}{3}$. Find the probability that either A or B will occur. **[2]**

i) $\frac{2}{15}$ ii) $\frac{8}{15}$

iii) $\frac{1}{15}$ iv) $\frac{3}{5}$

P.T.O.

d) The value of λ so that the vector field

$\bar{F} = (2x+3y)\hat{i} + (4y-2z)\hat{j} + (3x-\lambda 6z)\hat{k}$ is solenoidal is **[2]**

i) –6 ii) –1

iii) 1 iv) 0

e) $\nabla . \bar{r}$ is equal to

i) 2 ii) 1

iii) 0 iv) 3

f) The diff eq^n $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to condition $u(\infty, y) = 0$ with general Sol^n $u(x,y) = \left(C_1 e^{mx} + C_2 e^{-mx}\right)\left(C_3 \cos my + C_4 \sin my\right)$ the value of C_1 is **[2]**

i) 0 ii) $n\pi$

iii) 1 iv) π

Q2) a) Fit a straight line of the form $y = ax + b$ to the following data. **[5]**

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

b) The following marks have been obtained by a class of students in two papers.

Paper I	45	55	56	58	60	65	68	70	75	80	85
Paper II	56	50	48	60	62	64	65	70	74	82	90

Calculate the coefficient of correlation.

c) For the following distribution, find first four moments about the mean. **[5]**

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
f	4	36	60	90	70	40	10

OR

Q3) a) Fit a straight line to the following data. **[5]**

x	0	5	10	15	20	25
y	12	15	17	22	24	30

b) First four moments of a distribution about value 5 are, 2, 20, 40 and 50, find first four central moments, β_1 & β_2 **[5]**

c) From the record of correlation data, variance of x is 9 line of regressions are given by **[5]**

$8x-10y+66=0,\ 40x-18y=214$

Find

i) Mean values of x and y

ii) Coefficient of correlation between x and y

Q4) a) A can hit the target 1 out of 4 times, B can hit the target 2 out of 3 times, C can hit the target 3 out of 4 times. Find the probability the target is hit? **[5]**

b) An unbiased coin is thrown 10 times. Find the probability of getting atleast g Heads. **[5]**

c) In a certain examination test, 2000 students appeared in the subject of statistics. Average marks obtained were 50% with standard deviation 5%. How many students do you expect to obtain more than 60% of marks, supposing that marks follow normal distribution? **[5]**

(Given : Area corresponding to 2 is 0.4772)

OR

Q5) a) An envelope contains 6 tickets with numbers 1, 2, 3, 4, 5, 7. Another envelope contains 4 tickets with numbers 1, 3, 5, 7. An envelope is chosen at random and ticket is drawn from it. Find the probability that the ticket bears the numbers z or 5. **[5]**

b) The average number of misprints per page of a book is 1.5. Assuming the distribution of number of misprints to be poisson, find number of pages containing more than one misprint if the book contains 900 pages. **[5]**

c) The table below gives number of books issued from a certain library on the various days of a week. **[5]**

Days	No. of books issued
Mon.	120
Tues.	130
Wed.	110

Thurs.	115
Fri.	135
Sat.	110

Test at 5% l.o.s. whether issuing the book is day dependent (Given = $X^2_{5;0.05} = 11.07$)

Q6) a) Find the directional derivative of $\phi = 2x^2 + 3y^2 + z^2$ at the point (2, 1, 3) along the line $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z-3}{2}$ **[5]**

b) Show that vector field $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Also find corresponding scalar potential function ϕ such that $\overline{F} = \nabla\phi$ **[5]**

c) Evaluate $\int_c \overline{F} \cdot d\overline{r}$ for $\overline{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the curve $x = t, y = t^2, z = t^3$ from $t = 0$ to $t = 1$. **[5]**

OR

Q7) a) If the directional derivative of $\phi = axy + byz + cxz$ at (1, 1, 1) has maximum magnitude 4 in the direction parellel to X - axis, find the value of a, b, c. **[5]**

b) Show that (any one) **[5]**

i) $\nabla^4 (e^r) = e^r + \frac{4}{r} e^r$

ii) $\nabla\left(\frac{\overline{a} \cdot \overline{r}}{r^n}\right) = \frac{\overline{a}}{r^n} - \frac{n(\overline{a} \cdot \overline{r})}{r^{n+2}}$

c) Evaluate $\oint_c \overline{F} \cdot d\overline{r}$ using Green's theorem where $\overline{F} = (2x - \cos y)\hat{i} + x(4 + \sin y)\hat{j}$ and c is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ z = 0.$ **[5]**

Q8) a) Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ if **[7]**

i) $u\ (x, t)$ is bounded

ii) $u\ (0, t) = 0$

iii) $u\ (l, t) = 0$

iv) $u\ (x, 0) = \frac{u_0 x}{l}$ $0 < x < l$, u_0 is constant

b) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ **[8]**

i) $y\ (0, t) = 0$

ii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

iii) $y(\pi, t) = 0$

iv) $y(x, 0) = x,\ 0 \le x \le \pi$

OR

Q9) a) An infinitely long uniform plate is bounded by two parallel edges in the - y - direction and on end at right angles to them. The breadth of the plate is π. This end is maintained at temperature u_0 at all points, other edges as zero temperature. Find steady state temperature $u(x, y)$ If it satisfies $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}=0$ **[8]**

b) Use Fourier tansform to solve

$$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2} \quad 0<x<\infty$$

Under the condition -

i) $u(0, t) = 0 \qquad t > 0$

ii) $u(x,0)=\begin{cases}1 & 0<x<1\\ 0 & x>1\end{cases}$

c) $u(x, \text{t})$ is bounded. **[7]**