Total No. of Questions : 9]

SEAT No. :

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S.E. (Automobile & Mechanical/ Mechatronics/ Automation & Robotics/Mechanical S.W.)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III/IV) (207002)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No.1 is compulsory.*
2) *Solve Q.2 or Q.3, Q4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Choose the correct option.

a) The coefficient of Kurtosis β_2 is given by **[1]**

i) $\dfrac{\mu_4}{\mu_3}$ ii) $\dfrac{\mu_4}{\mu_2^2}$

iii) $\dfrac{\mu_3}{\mu_2^2}$ iv) $\dfrac{\mu_3^2}{\mu_2^3}$

b) From the given information $\Sigma x = 235$, $\Sigma x^2 = 6750$, $n = 10$ then standard deviation of x is **[2]**

i) 11.08 ii) 13.08

iii) 8.08 iv) 7.6

c) If x follows the binomial distribution with parameter n and $p = ½$ and $p(x = 5) = p(x = 4)$, then $p(x = 2)$ is **[2]**

i) ${}^{7}C_2\left(\dfrac{1}{2}\right)^7$ ii) ${}^{11}C_2\left(\dfrac{1}{2}\right)^{11}$

iii) ${}^{10}C_2\left(\dfrac{1}{2}\right)^{10}$ iv) ${}^{9}C_2\left(\dfrac{1}{2}\right)^9$

P.T.O.

d) A unit vector normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 1) is ____ **[2]**

i) $\frac{1}{\sqrt{6}}(\bar{i} + 2\bar{j} + \bar{k})$

ii) $\frac{1}{\sqrt{3}}(2\bar{i} + 4\bar{j} + 4\bar{k})$

iii) $\bar{i} + 2\bar{j} + 2\bar{k}$

iv) $2\bar{i} + 2\bar{j} - \bar{k}$

e) The most general solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ representing metal plate having length x and breadth $y \to \infty$ is **[2]**

i) $(c_1 \cos mx + c_2 \sin mx)(c_3 e^{my} + c_4 e^{-my})$

ii) $(c_1 e^{-my} + c_2 e^{my})$

iii) $(c_1 e^{mx} + c_2 e^{-mx})(c_3 \cos my + c_4 \sin my)$

iv) $(c_1 e^{-mx} + c_2 e^{mx})$

f) The value of $\nabla\left(\frac{1}{r}\right) =$ ________ **[1]**

i) $-\frac{\bar{r}}{r^2}$

ii) $\frac{\bar{r}}{r}$

iii) $-\frac{\bar{r}}{r^3}$

iv) 0

Q2) a) The first four moments about the value 3.5 are 0.058064, 0.451612, 0.082259 and 0.5 calculate the first four central moments. **[5]**

b) Compute correlation coefficient for the following data **[5]**

x	152	158	169	182	160	166	182
y	198	178	167	152	180	170	162

c) Fit a straight line for the following data of the form $y = ax + b$ **[5]**

x	5	4	3	2	1
y	1	2	3	4	5

OR

Q3) a) If $\Sigma f = 27$, $\Sigma fx = 91$, $\Sigma fx^2 = 359$, $\Sigma fx^3 = 1567$, $\Sigma fx^4 = 7343$. **[5]**

Find first four moments about origin.

b) Find the regression line of y on x from the data **[5]**

x	65	63	67	64	68	62	70	66	68	67
y	68	66	68	65	69	66	68	65	71	67

c) For the tabulated values of x and y fit a straight line of the form $y = mx+c$. **[5]**

x	1.0	3.0	5.0	7.0	9.0
y	1.5	2.8	4.0	4.7	6.0

Q4) a) A box containing 6 red balls, 4 white balls and 5 blue balls. Three balls are drawn successively from the box. Find the probability that they are drawn in the order red, white and blue if each ball is not replaced. **[5]**

b) A random variable X B(n = 6, P). Find P if 9P(R = 4) = 4P(R = 2). **[5]**

c) In a certain city 4,000 tube lights are installed. If the lamps have average life of 1,500 burning hours. Assuming normal distribution **[5]**

i) How many lamps will fail in first 1,400 hours?

ii) How many lamps will last beyond 1,600 hours?

Given A = 0.3413 for Z =1.

OR

Q5) a) A coin thrown 4 times successively. X denotes the number of heads. Find the expectation of X. **[5]**

b) A manufacturer of electronic goods has 4% of his product defective. He sells the articles in packet of 300 and guarantees 90% good quality. Determine the probability that a particular pocket will violate the guarantee. **[5]**

c) A set of five similar coins in tossed 210 times and the result is **[5]**

No of heads	0	1	2	3	4	5
Frequency	2	5	20	60	100	23

Test the hypothesis that the data follow a binomial distribution. [Given $\chi^2_{5,0.05} = 11.070$]

OR

Q6) a) The position vector of a particle at time t is $\bar{r} = \cos(t-1)\bar{i} + \sin h(t-1)\bar{j} + mt^3\bar{k}$. Find the condition imposed on m by requiring that at time t =1, the acceleration is normal to the position vector. **[5]**

b) Show that $\bar{F} = (ye^{xy}\cos z)\bar{i} + (xe^{xy}\cos z)\bar{j} - e^{xy}\sin z\bar{k}$ is irrotational. Find scalar potential function & such that $\bar{F} = \nabla\phi$. **[5]**

c) Evaluate $\int_c \bar{F}\cdot d\bar{r}$ where $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the straight line (0, 0, 0) and (2, 1, 3). **[5]**

Q7) a) Find the directional derivative of $\varphi = e^{2x}\cos yz$ at (0, 0, 0) in the direction of tangent to the curve $x = a\sin t$, $y = a\cos t$, $z =$ at at $t = \pi/2$. **[5]**

b) Solve any one : **[5]**

i) Show that $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})}{r^{n+2}}\bar{r}$

ii) Show that $\bar{b}\times\nabla[\bar{a}\cdot\nabla\log r] = \frac{\bar{b}\times\bar{a}}{r^2} - \frac{2(\bar{a}\cdot\bar{r})}{r^4}(\bar{b}\times\bar{r})$

c) Evaluate $\iint_s \nabla\times\bar{\text{F}}\cdot d\bar{s}$ where $\bar{\text{F}} = y\bar{i} + z\bar{j} + x\bar{k}$ where s is surface of the paraboloid. $z = 1 - x^2 - y^2$, $z \geq 0$. **[5]**

Q8) a) Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ subject to **[8]**

i) $u(o, t) = 0, \ \forall t$

ii) $u(l, t) = 0, \ \forall t$

iii) $u(x, t)$ is bounded

iv) $u(x, o) = \frac{uox}{l}$, for $o \le x \le l$.

b) A string is stretched and fastened to two points l a part, motion is streatched by displaying the string in the form $u = a\sin\left(\frac{\pi x}{l}\right)$ from rest it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. **[7]**

OR

Q9) a) An infinitely long plane uniform plate is bounded by two parallel edges in the y - direction and an end at right angles to them. The breadth of the plate is π. This end is maintained at temperature u_0 (constant) at all points and other edges at zero temperature. Find the steady - state temperature $u(x, y)$. **[8]**

b) Use fourier sine transform to solve **[7]**

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to

i) $u(0, t) = 0, \ \forall t > 0$

ii) $u(x, 0) = 1, \quad 0 < x < 1$

$\quad = 0 \quad x > 0$

iii) $u(x, t)$ is bounded.

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