Total No. of Questions : 9]

SEAT No. :

PD-4167

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[6402]-128
S.E. (Automobile/Mechanical) (Mechanical Sandwich)
ENGINEERING MATHEMATICS - III
(2019 Pattern) (Semester - IV) (207002)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Q. 1 is compulsory.***
2) ***Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Figures to the right indicate full marks.***
5) ***Use of electronic pocket calculator is allowed.***
6) ***Assume suitable data, if necessary.***

Q1) **Choose the correct option.**

i) Mean and variance of binomial distribution are 6 and 2 respectively. The value of p is **[2]**

a) $\frac{1}{3}$ b) $\frac{1}{2}$

c) $\frac{2}{3}$ d) 1

ii) The curve is given by $\bar{r} = t\,\bar{i} + 2t\,\bar{j} + t^2\bar{k}$ then tangent vector to the curve at t = 1 is **[2]**

a) $\bar{i} + 2\bar{j} + 2\bar{k}$ b) $\bar{i} - 2\bar{j} + 2\bar{k}$

c) $\bar{i} + 2\bar{j} - 2\bar{k}$ d) $\bar{i} - 2\bar{j} - 2\bar{k}$

iii) Coefficient of variation of the data 1,3,5,7,9 is **[2]**

a) 54.23 b) 56.57

c) 55.41 d) 60.19

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iv) The most general solution u(*x*,t) of the one dimention heat equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$; is **[2]**

a) A cos (*kx*) sin (kt)

b) A sin (*kx*) cos (kt)

c) $(C_1 \cos mx + C_2 \sin mx).e^{-km^2t}$

d) $(C_1 \cos mx).e^{-km^2t}$

v) For the data presented in the form of frequency distribution, variance (σ^2) is given by. **[1]**

a) $\frac{\sum fx}{N}$ b) $\frac{\sum f|x-\bar{A}|}{N}$

c) $\frac{\sum f(x-\bar{x})^2}{N}$ d) $\sqrt{\frac{\sum f(x-\bar{x})^2}{N}}$

vi) Vector field $\bar{F}$ is solenoidal if **[1]**

a) $\nabla \times \bar{F} = 0$ b) $\nabla . \bar{F} = 0$

c) $\nabla^2 \bar{F} = 0$ d) $\nabla \times \nabla = 0$

Q2) a) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. **[5]**

b) Fit a curve $y = ax^6$ using following data **[5]**

x	2000	3000	4000	5000	6000
y	15	15.5	16	17	18

c) Obtain correlation cofficient between population density (Per square miles) and death rate (Per thousand persons) from data related to 5 cities. **[5]**

Population density	200	500	400	700	800
Death rate	12	18	16	21	10

OR

Q3) a) The scores obtain by two batsman A & B in 10 matches are given below. Determine who is more consistant and who is batter run gether? **[5]**

Batsman A	30	44	66	62	60	34	80	46	20	38
Batsman B	34	46	70	38	55	48	60	34	45	30

b) A simply supported beam carries a concentrated Load p(kg) its middle point. Corresponding to various values of P, the maximum deflection Y cms is tabulated as : **[5]**

P	100	120	140	160	180	200
Y	0.90	1.10	1.20	1.40	1.60	1.70

Find a law of the form y = aP + b by using least square method.

c) Determine the equation of regression lines for the following data. **[5]**

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

and obtain an estimate of *y* for *x* = 4.5.

Q4) a) A has 2 tickets in a lottery in which there are 3 prizes and 5 blanks; B has 3 tickets in a lottery in which there are 4 prizes and 6 blanks. Show that A' chance of success is to B's as 27:35. **[5]**

b) The number of industrial injuries per week in a particular factory follows a poisson distribution with mean 0.5. Find the probability that during a week, there will be at the most one injury. **[5]**

c) The height of students in a class follows a normal distribution with mean 190 cm and variance 80 cm^2. Among the 1,000 students from the school, how many are expected to have height above 200 cm? **[5]**

[Given : z = 1.118, A = 0.3686]

OR

Q5) a) An urn contain 10 white and 3 black balls, white another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then ball is drawn from the latter. What is the probability that it is a white ball? **[5]**

b) A fair coin is tossed 5 times. What is the probability of getting at least two heads? **[5]**

c) The theory predicts the proportion of bean in the four groups A,B,C and D should be 9:3:3:1. In an experiment among 1600 beans, The numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? [Given $\chi^2_{3,0.05} = 7.815$] **[5]**

Q6) a) Find the angle between tangents to the curve **[5]**

$\bar{r} = t^3\,\bar{i} + 4t\,\bar{j} + 2t^2\,\bar{k}$ at $t = 0, t = 1$

b) Show that $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - xz)\,\bar{j} + (z^2 - xy)\bar{k}$ is irrotational. Find scalar potential. ϕ such that $\bar{F} = \nabla\,\phi$. **[5]**

c) Evaluate $\int_C \bar{F}.\,d\bar{r}$ where $\bar{F} = x^2\,\bar{i} + 2xy\,\bar{j} + z\bar{k}$ where C is the straight line joining (0,0,2) to (3,1,1). **[5]**

OR

Q7) a) Show that $\bar{F} = (2xz^3 + 6y)\bar{i} + (6x - 2yz)\,\bar{j} + (3x^2z^2 - y^2)\bar{k}$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$. **[5]**

b) Solve any one **[5]**

i) Show that $\bar{b} \times \nabla[\bar{a}.\nabla \log r]$

$$= \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2(\bar{a}.\bar{r})}{r^4}(\bar{b} \times \bar{r})$$

ii) Show that $\nabla^2 (r^5 \log r) = r^3 (30 \log r + 11)$

c) Use Stoke's theorem evaluate **[5]**

$\iint_S (\nabla \times \bar{F}).\bar{ds}$ where $\bar{F} = 3y\,\bar{i} - xz^2\,\bar{j} + yz\bar{k}$ S is surface of paraboloid $2z = x^2 + y^2$ bounded by $z = 2$.

Q8) a) The temperature at any point of the insulated metal rod of one meter length $l = 1$ is governed by the differential equation **[8]**

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2},$$

Find the temperature u(x,t) subject to the following conditions

i) $u(0,t) = 0°c \forall$ t,

ii) $u(l,t) = 0°c \forall$ t,

iii) $u(x,0) = 40°c$

b) A string is stretched and fastened to two points l a part, motion is streatched by displaying the string in the form of $u = a \sin\left(\frac{\pi x}{l}\right)$ from which it is released at time t = 0. Find the displacement $u(x,t)$ from one end . **[7]**

OR

Q9) a) An infinitely long uniform plate is bounded by parallel edges in y - direction and an end at right angles to them. The breath of the plate is π. This end is maintained at the constant temperature 40°c ($u(x,0) = 40$, $0 < x < \pi$) at all points and other edges at zero temperature. Find the steady state temperature u(x,y). **[8]**

b) Using Fourier sine transform solve the partial differential equation

$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$ subject to **[7]**

i) $u\,(0,t) = 0$, $t > 0$,

ii) $u\,(x,0) = e^{-x}$, $x > 0$,

iii) u and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$