Total No. of Questions : 9]

SEAT No. :

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S.E. (Automobile & Mechanical Engineering)/(Mechanical Sandwich)/ (Automation & Robotics Engineering)/(Mechatronics Engineering)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - IV) (207002)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question 1 is compulsory.*
2) *Solve Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicates full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Choose the correct option :

a) If the data presented in the form of frequency distribution then the arithmatic mean $\overline{x}$ is given by **[1]**

i) $\frac{\sum fx}{N}$

ii) $\frac{1}{N}\sum f\left|x-\overline{x}\right|$

iii) $N\sum fx$

iv) $\frac{\sum fx^2}{N}$

b) From the given information standard deviation of $x = 4$, standard deviation of $y = 1.8$, coefficient of regression of y on x is 0.32. The coefficient of correlation is **[2]**

i) 0.711

ii) 0.622

iii) 0.743

iv) 0.543

c) The mean and variance of binomial probability destribution are 36 and 3 rspectively. Number of trials n is given by **[2]**

i) 42

ii) 36

iii) 48

iv) 24

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d) The tangent vector to the curve $\bar{r} = t\bar{i} + t^2\bar{j} + t^3\bar{k}$ at $t = 1$ is ________. **[2]**

i) $\bar{i} + \bar{j} + \bar{k}$

ii) $\bar{i} + 2\bar{j} + 3\bar{k}$

iii) $\bar{i} - 2\bar{j} + 3\bar{k}$

iv) $\bar{i} - 2\bar{j} - 3\bar{k}$

e) Solution of $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ representating the vibration of string of length l fined at end points with initial and boundary conditions **[1]**

I) $y(0, t) = 0$

II) $y(l, t) = 0$

III) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

IV) $y(x, 0) = f(x)$

The most general solution $y(x, t)$ is **[1]**

i) $(c_1 \cos mx + c_2 \sin mx)(c_3 \cos cmt + c_4 \sin cmt)$

ii) $(c_1 e^{mx} + c_2 e^{-mx})$

iii) $(c_1 e^{-my} + c_2 e^{-my})$

iv) None of these

f) The value $\nabla \cdot \bar{r} =$ ________ **[1]**

i) 0

ii) 1

iii) 2

iv) 3

Q2) a) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Obtain the First Four central moments, also find β_1 and β_2. **[5]**

b) Obtain correlation coefficient for the following data **[5]**

x	200	500	400	700	800
y	12	18	16	21	10

c) Fit a straight line of the form $y = mx + c$ for the following data **[5]**

x	0	1	2	3	4	5	6	7
y	–5	–3	–1	1	3	5	7	9

OR

Q3) a) The first four moments about the value 25 are –1.1, 89, –110 and 23300. Calculate the first four central moments. **[5]**

b) Obtain regression lines for the following data **[5]**

x	6	2	10	4	8
y	9	11	5	8	7

c) Use least square method to fit a straight line for the following data. **[5]**

x	0	2	4	6	8	12	20
y	10	12	18	22	20	30	30

Q4) a) A can hit the target 1 out of 4 times, B can hit the target 2 out of 3 times, c can hit the target 3 out of 4 times. Find the probability of at least two hit the target. **[5]**

b) On an average a box containing 10 articles in Likely to have 2 defectives. If we consider a consignment of 100 boxes. How many of them are expected to have three or less defectives? **[5]**

c) In a certain factory turning out razor blades, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use poisson distribution to calculate the approximate number of packets containing no defective and two defective blades, in a consignment of 10,000 packets. **[5]**

OR

Q5) a) Variable x takes the value 0, 1, 2, 3, 4, 5 with probability of each as $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{3}{15}, \frac{5}{15}, \frac{1}{15}$. Find expectation of x. **[5]**

b) The mean weight of 500 students is 63 kgs and the standard deviation is 8 kgs. Assuming that the weights are normally destributed, find how many students weight 52 kgs? **[5]**

[Given at $z_1 = +1.44 = A_1 = 0.4049$ and $z_2 = +1.13 = A_2 = 0.4251$]

c) A set of five similar coxins are tossed 210 times and the result is

No of heads	0	1	2	3	4	5
Frequency	2	5	20	60	100	23

Test the hypothesis that the data follow a bionomial destribution. Given $x^2_{5,0.05} = 11.07$. **[5]**

Q6) a) Find the angle between tangent to the curve $x = t^2+1$, $y = t^2-1$, $z = t$ at $t = 1$, $t = 2$. **[5]**

b) Show that $\bar{F} = \left(y^2 \cos x + z^2\right)\bar{i} + (2y\sin x)\bar{j} + 2zx\bar{k}$ is irrotational. Find scalar potential function ϕ. Such that $\bar{F} = \nabla\phi$. **[5]**

c) If $\bar{F} = \left(2xy + 3z^2\right)\bar{i} + \left(x^2 + 4yz\right)\bar{j} + \left(2y^2 + 6zx\right)\bar{k}$ evaluate $\int_C \bar{F}.d\bar{r}$ where c is curve $x = t$, $y = t^2$, $z = t^3$ joining (0, 0, 0) & (1, 1, 1). **[5]**

OR

Q7) a) Find the directional derivative of $\phi = x^2yz^3$ at (2, 1, –1) along the vector $i - \bar{j} + \bar{k}$. **[5]**

b) Solve any one. **[5]**

i) Show that $\nabla^2\left[\nabla\cdot\left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}$.

ii) If $\wp\,\bar{E} = \nabla\phi$ prove that $\bar{E}\cdot \text{curl}\,\bar{E} = 0$.

c) Evaluate $\iint_s (\nabla\times F)\cdot d\bar{s}$ where $\bar{F} = \left(x^3 - y^3\right)\bar{i} - xyz\bar{j} + y^3\bar{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the surface $x = 0$. **[5]**

Q8) a) A homogeneous rod of conducting material of length 10 cm has its ends kept at zero temperature and the temperature initially is **[8]**

$u(x, 0) = x, \quad 0 \le x \le 50$

$= 100-x, \quad 50 \le x \le 100$

Find the temperature $u(x, t)$ at any time t.

b) A tightly streatched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_o \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end and at any time t. **[7]**

OR

Q9) a) A rectangular plate with insulated surfaces is 10 cm wide and so long compaired to its width that it may be considered infinite in length without introducing an appreciable error. If temperature along short edge $y = 0$ is given **[8]**

$$u(x, 0) = 100 \sin\left(\frac{\pi x}{10}\right),\ 0 \le x \le 10$$

While two edges $x = 0$ and $x = 10$ as well as other short edge are kept at 0ºC - find the steady state temperature $u(x, y)$.

b) Use Fourier sine transform to solve **[7]**

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2},\ 0 < x < \infty,\ t > 0 \text{ subject to}$$

i) $u(0, t) = 0$

ii) $u(x, 0) = e^{-x},\ x > 0$

iii) u and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$

x x x