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S.E. (Automobile&Mechanical/Mechatronics) (Mechanical S.W) (Automation & Robotics)

ENGINEERING MATHEMATICS-III

(2019 Pattern) (Semester-IV) (207002)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Q.1 is compulsory.***
2) ***Solve Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Figures to the right indicate full marks.***
5) ***Use of electronic pocket calculator is allowed.***
6) ***Assume suitable data, if necessary.***

Q1) Choose the correct option.

a) ∇e^r is equal to **[2]**

i) $e^r \bar{r}$ ii) $\frac{e^r}{r}$

iii) $\frac{e^r}{r}\bar{r}$ iv) $\frac{r}{e^r}\bar{r}$

b) The most general solution of heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to usual initial and boundary conditions is u= **[2]**

i) $(c_1 \cos mx + c_2 \sin mx)e^{-c^2 m^2 t}$

ii) $(c_1 \cos mx)e^{-c^2 m^2 t}$

iii) $(c_2 \sin mx)e^{-c^2 m^2 t}$

iv) None of these

c) For the data presented in the form of frequency distribution, then the arithmetic mean $\bar{x}$ is given by (by considering N=Σf) **[1]**

i) $\frac{\sum fx}{N}$ ii) $N\sum fx$

iii) $\frac{\sum f|x-A|}{N}$ iv) $\frac{\sum fx^2}{N}$

P.T.O.

d) The standard deviation and arithmetic mean of the distribution are 12 and 45.5 respectively. Coefficient of variation of the distribution is **[2]**

i) 26.37 ii) 32.43

iii) 12.11 iv) 22.15

e) An unbised coin is thrown 10 times, the probability of getting 6 heads is **[2]**

i) 0.2000 ii) 0.2050

iii) 0.5 iv) 0.3

f) The value of $\nabla.\bar{r}, \bar{r} = xi + yj + zk,$ is **[1]**

i) 0 ii) 1

iii) 2 iv) 3

Q2) a) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about the mean. Also evaluate β_1, β_2 and Comment upon the Skewness and Kurtosis of the distribution. **[5]**

b) For the tabulated values of x and y given below fit a linear curve of the type $y = mx + c$ **[5]**

x	1.0	3.0	5.0	7.0	9.0
y	1.5	2.8	4.0	4.7	6.0

c) The regression equations are $8x - 10y + 66 = 0$ and $40x + 18y = 214$. The value of variance of x is 9. Find **[5]**

i) The mean values of x and y.

ii) The correlation x and y.

iii) The standard deviation of y.

OR

Q3) a) Goals scored by two teams A and B in a football season were as follows: **[5]**

No of Goals Scored in a match		0	1	2	3	4
No of Matches	A	27	9	8	5	4
	B	17	9	6	5	3

b) Using following values of x and y. Fit the curve of the type $y = ab^x$ using least square method. **[5]**

x	2.1	2.5	3.1	3.5	4.1
y	5.14	6.788	10.29	13.58	20.578

c) Obtain regression lines for the following data **[5]**

x	2	3	5	7	9	10	12	15
y	2	5	8	10	12	14	15	16

Find estimal of y when x=6

Q4) a) Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that they are both king. If **[5]**

i) The first card drawn is replaced.

ii) First card drawn is not replaced.

b) The number of accidents per week on a highway follows a poisson distribution with mean 0.5. Find the probability that during a week there will be at the most one accident. **[5]**

c) The lifetime of an article has a normal distribution with mean 400 hours and standard deviation 50 hours. Assuming normal distribution. Find the expected number of articles out of 2000 whose lifetime lies between 335 hours to 465 hours [Given : z=1.3, A=0.4032] **[5]**

OR

Q5) a) A can hit target 1 out of 4 times, B can hit the target 2 out of 3 times, C can hit the target 3 out 4 times. Find the probability of at least two hit the target. **[5]**

b) A fair coin is tossed 5 times. What is the probability of getting at least two tails? **[5]**

c) A nationalized bank utilizes four teller windows to render fast service to the customers. on a particular day 800 customers were observed. They were given service at the different windows as follows. **[5]**

Window Number	1	2	3	4
Number of Customers	150	250	170	230

Test whether the customers are uniformly distribution over the windows.

$\left[Given\ \chi^2_{3.005} = 7.815\right]$

Q6) a) Find the angle between the tangents to the curve $\bar{r} = t\bar{i} + t^2\bar{j} + t^3\bar{k}$ at t=1 and $t = -1$ **[5]**

b) Find $f(r)$ so that $f(r)\,\bar{r}$ is solenoidal. **[5]**

c) Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (2x + y)\bar{i} + (3y - x)\bar{j}$ where C is the curve along Straight line joining (0,0) and (3,2). **[5]**

OR

Q7) a) Find the directional derivative of $\varphi = 5x^2y - 5y^2z + 2z^2x$ at (1,1,1) in the direction of line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$ **[5]**

b) Solve any one: **[5]**

i) Show that $\nabla\left(\frac{\bar{a}.\bar{r}}{r^3}\right) = \frac{\bar{a}}{r^3} - \frac{3(\bar{a}.\bar{r})}{r^5}\bar{r}$

ii) Show that $\nabla^2\left(\frac{1}{r}\log r\right) = -\frac{1}{r^3}$

c) Apply Green's theorem to evaluate.

$\int_c (2x^2 - y^2)\,dx + (x^2 + y^2)dy$ Where C is the curve of area enclosed by the axis and the upper half of the circle $x^2 + y^2 = 16$. **[5]**

Q8) a) Solve $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ subject to the following conditions. **[8]**

i) u is finite for all t

ii) $u(0,t) = 0, \forall t$

iii) $u(l,t) = 0, \forall t$

iv) $u(x,0) = u_0(\text{constant}), \text{for } 0 \le x \le l$

Where l is the length of the bar.

b) A tightly streatched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t. **[7]**

OR

Q9) a) An infinitely uniform metal plate is enclosed between lines $y=0$ and $y=l$, for $x>0$. The temperature is zero along the edges $y=0$ and $y=l$ and at infinity. If the edge $x=0$ is kept at a constant temperature u_0, find the temperature distribution u(x,y) **[8]**

b) Use fourier sine transform to solve partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < \infty, t > 0$$ **[7]**

subjected to

i) $u(0,t) = 0, \quad \forall t$

ii) $u(x,0) = 4, \ 0 < x < 1$

$=0, \quad x > 1$

iii) u (x,t) is bounded.

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