Total No. of Questions : 9]

SEAT No. :

P1582

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[6002]-212

S.E. (Automobile/ Mechanical/ Mechanical Sandwich/ Mechatronics/ Automation & Robotics)

ENGINEERING MATHEMATICS -III

(2019 Pattern) (Semester - III & IV) (207002)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No.1 is compulsory.*
2) *Solve Q.No.2 or Q.No.3, Q.No.4 or Q.No.5, Q.No.6 or Q.No.7, Q.No.8 or Q.No.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Choose correct option of the following.

a) The curl of vector field $\overline{F} = x^2 y\,\overline{i} + xyz\,\overline{j} + z^2 y\,\overline{k}$ at point (0,1,2) is ____. **[2]**

i) $4\overline{i} - 2\overline{j} + 2\overline{k}$

ii) $4\overline{i} + 2\overline{j} + 2\overline{k}$

iii) $4\overline{i} + 2\overline{k}$

iv) $2\overline{i} + 4\overline{k}$

b) The most general solution of wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ subjected to usual initial and boundary conditions **[2]**

I) $u(0,t) = 0, \forall t,$

II) $u(l,t) = 0, \forall t,$

III) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

IV) $u(x, 0) = f(x)$ is $u(x, t) =$

i) $(C_1 \cos mx + C_2 \sin mx)(C_3 \cos cmt + C_4 \sin cmt)$

ii) $(C_1 \cos mx + C_2 \sin mx)$

iii) $C_3 \cos cmt + C_4 \sin cmt$

iv) $C_1 e^{mx} + C_2 e^{-mx}$

P.T.O.

c) In a Poisson distribution if $n = 100, p = 0.01$ then $P(r = 0) =$ ________ [2]

i) $\frac{1}{e}$ ii) $\frac{2}{e}$

iii) $\frac{3}{e}$ iv) $\frac{4}{e}$

d) The first four moments about value 4 are 0, 2, 0, 11; then the value of fourth central moment is [2]

i) 0 ii) 2

iii) 11 iv) $\frac{2}{11}$

e) The Regression line of y on x is given by [1]

i) $(x-\overline{x}) = b_{xy}(y-\overline{y})$ ii) $(y-\overline{y}) = b_{yx}(x-\overline{x})$

iii) $(x-\overline{x}) = b_{yx}(y-\overline{y})$ iv) $(y-\overline{y}) = b_{xy}(x-\overline{x})$

f) The value of $\nabla e^r =$ ________ [1]

i) $e^r \overline{r}$ ii) $\frac{e^r}{r}$

iii) $\frac{e^r}{r}\overline{r}$ iv) $\frac{r}{e^r}\overline{r}$

Q2) a) Fit a straight line $y = mx + c$ to the following data. [5]

x	5	4	3	2	1
y	1	2	3	4	5

b) First four moments of a distribution about the value 2 are 1, 2.5, 5.5 and 16. Find first four central moments β_1 and β_2. [5]

c) Calculate the coefficient of correlation for the following data. [5]

x	–1	1	2	4	6	7
y	–1	2	3	3	5	8

OR

Q3) a) Fit a straight line of the form $y = ax + b$ to the following data. **[5]**

x	0	1	2	3	4
y	14	27	40	55	68

b) The first four moments of distribution about working mean 3.5 are 0.0375, 0.4546, 0.0609 and 0.5074. Calculate the first four central moments. **[5]**

c) Obtain regression line of the following data. **[5]**

x	2	4	5	6	8	11
y	18	12	10	8	7	5

Q4) a) A series of five one day matches is to be played between India and Sri Lanka. Assuming that the probability of India's win in each match as 0.6 and result of all five matches is independent of each other. Find the probability that India wins the series. **[5]**

b) The number of breakdowns of a computer in a week is a Poisson variable with $\lambda = np = 0.3$. What is the probability that the computer will operate **[5]**

i) with no breakdown

ii) at most one breakdown in a week.

c) The life time of a certain component has a normal distribution with mean of 400 hours and standard deviation of 50 hours. Assuming a normal sample of 1000 components, find number of components whose life time lies between 340 to 465 hours [Given : A(z =1.2)= 0.3849, A (z = 1.3)= 0.4032]. **[5]**

OR

Q5) a) A coin is tossed 4 times. X denote the number of heads. Find the expectation of x. **[5]**

b) If 10% of the rivet's produced by the machine are defective, find the probability that out of 5 rivets chosen at random at least two will defective. **[5]**

c) A die is tossed 300 times gave the following result.

Score	1	2	3	4	5	6
Frequency	43	49	56	45	66	41

Are the data consistent at 5% level of singificance with hypothesis that the die is unbiased? (Given : $\chi^2_{5,0.05} = 11.07$). **[5]**

Q6) a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the point (2, –1, 2). **[5]**

b) Show that $\bar{F} = (ye^{xy} \cos z)\,\bar{i} + (xe^{xy} \cos z)\,\bar{j} - e^{xy} \sin z\,\bar{k}$ is irrotational. Find scalar potential function ϕ such that $\bar{F} = \nabla\phi$. **[5]**

c) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = x^2\bar{i} + xy\bar{j}$ where C is arc of parabola $y = x^2$ joining (0, 0) & (1, 1). **[5]**

OR

Q7) a) Find the directional derivative of $\phi = xyz$ in the direction normal to the surface $x^2y + xy^2 + yz^2$ at (1,1,1). **[5]**

b) Solve any one. **[5]**

i) Prove that $\nabla^2\left[\nabla\cdot\left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}$

ii) Prove that $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^3}\right) = \frac{\bar{a}}{r^3} - 3\frac{(\bar{a}\cdot\bar{r})}{r^5}\bar{r}$

c) Evaluate by using Green's theorem $\int_C (\cos x \sin y - 4y)\,dx + \sin x \cos y\,dy$ where C is $x^2 + y^2 = 1$. **[5]**

Q8) a) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is **[8]**

$u(x, 0) = x, \ 0 \le x \le 50$

$\qquad = 100 - x, \ 50 \le x \le 100.$

Find the temperature $u(x, t)$ at any time t.

b) A string is stretched and fastened to two points l apart, motion is stretched by displaying the string in the form $u = a \sin\left(\frac{\pi x}{l}\right)$ from which is released at time $t = 0$. Find the temperature distribution $u(x, t)$ from one end. **[7]**

OR

Q9) a) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = l$, for $x > 0$. The temperature is zero along the edges $y = 0$ and $y = l$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature distribution $u(x, t)$. **[8]**

b) Use Fourier sine transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t > 0.$

Subjected to **[7]**

i) $u(0, t) = 0, \ \forall \ t > 0.$

ii) $u(x, 0) = 1, 0 < x < 1$

$= 0, x > 1$

iii) $u(x, t)$ is bounded.

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