Total No. of Questions : 9]

SEAT No. :

**PA-1285**

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**[5925]-311**

**S.E. (Automobile & Mechanical/Automation & Robotics/ Mechatronics /Mechanical /Mechanical Sandwich)**

# ENGINEERING MATHEMATICS - III
## (2019 Pattern) (Semester - IV) (207002)

*Time : 2½ Hours]* *[Max. Marks : 70*

*Instructions to the candidates:*

1) *Question No. 1 is compulsory. Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Neat diagrams must be drawn whenever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

***Q1)*** a) The first four moments of a distribution about mean of the variable are 0, 2, 0 and 11. Then $\beta_2$ = **[2]**

i) 2.5 ii) 2.3999

iii) 2.75 iv) 0.5987

b) If $\overline{F} = (x^2y)\hat{i} + (xyz)\hat{j} + (z^2y)\hat{k}$ then curl $\overline{F}$ at (1, 1, 2) is **[2]**

i) $5\hat{i} + \hat{j}$ ii) $3\hat{i} + \hat{j} + \hat{k}$

iii) $3\hat{i} + \hat{k}$ iv) $3\hat{i} + \hat{j}$

c) The most general solution of the partial differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ representing heat flow along a bar is **[2]**

i) $(c_1 \cos mx + c_2 \sin mx)e^{-c^2m^2t}$

ii) $(c_1 \cos mx + c_2 \sin mx)e^{-m^2t}$

iii) $(c_1 \cos mx + c_2 \sin mx)(c_3 \cos cmt + c_4 \sin cmt)$

iv) $(c_1 \cos mx + c_2 \sin mx)(c_3e^{my} + c_4e^{-my})$

d) In Binomial probability distribution, if $p = q$, then $P(\overline{X} = r)$ is **[2]**

i) ${}^nc_r\left(\frac{1}{2}\right)^{n-r}$ ii) ${}^nc_1\left(\frac{1}{2}\right)^n$

iii) ${}^nc_r\left(\frac{1}{2}\right)^n$ iv) ${}^nc_n\left(\frac{1}{2}\right)^n$

*P.T.O.*

e) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $\nabla \cdot \bar{r} =$ **[1]**

i) 1 ii) 2

iii) 3 iv) 4

f) In a poisson distribution if $P(r=3) = 6P(r=4)$, then $P(r=2)$ is equal to **[1]**

i) 0.025 ii) 0.01148

iii) 0.251 iv) 0.1148

***Q2)*** a) Fit a straight line of the Form $y = ax + b$ to the following data. **[5]**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 1 | 3 | 4 | 5 | 6 | 8 |
| $y$ | –3 | 1 | 3 | 5 | 7 | 11 |

b) Calculate the first four moments about the mean of the following distribution. **[5]**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| F | 6 | 15 | 23 | 42 | 62 | 60 | 40 | 24 | 13 | 5 |

c) Find the coefficient of correlation for the following table. **[5]**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 10 | 14 | 18 | 22 | 26 | 30 |
| $y$ | 18 | 12 | 24 | 6 | 30 | 36 |

OR

***Q3)*** a) Fit a straight line to the following data. **[5]**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 0 | 5 | 10 | 15 | 20 | 25 |
| $y$ | 12 | 15 | 17 | 22 | 24 | 30 |

b) First four moments of a distribution about value 4 are –1.5, 17, –30 and 108. Find the first four moments about mean $\beta_1$ & $\beta_2$. **[5]**

c) Obtain the regression lines for the following table. **[5]**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 6 | 2 | 10 | 4 | 8 |
| $y$ | 9 | 11 | 5 | 8 | 7 |

***Q4)*** a) From 20 tickets marked 1 to 20, one ticket is drawn at random. Find the probability that it is marked with multiple of 3 or 5. **[5]**

b) A fair coin is tossed 6 times. Find a probability of getting: **[5]**

i) at least four heads

ii) not heads

c) Assuming that the distance of 1000 brass plugs taken consecutively from machine from a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm. How many of the plugs are likely to be approved if the acceptable diameter is $0.752 \pm 0.004$ cm. (Given Area = 0.478 for $z$ = 2.25 and Area 0.4599 for $z$ = 1.75). **[5]**

OR

*Q5)* a) A can hit the target 1 out of 4 times. B can hit 2 out of 3 times. C can hit the target 3 out of 4 times. Find the probability that at least 2 hit the target. **[5]**

b) In a certain factory turning out razor blades there is a small chance of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a pack of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective and two defective blades, in a consignment of 10,000 packets. **[5]**

c) Among 64 off spring of a certain cross between European horses, 34 were red, 10 were black and 20 were white. According to a genetic model, these numbers should be in the ratio 9 : 3 : 4. Is the data consistant with the model at 5% level of significance $\left(\chi^2_{v-1,0.05} = 5.99\right)$. **[5]**

*Q6)* a) Find the directional derivative of $\phi = x^2 - y^2 - 2z^2$ at the point $P(2,-1,3)$, in the direction PQ where Q is (5, 6, 4). **[5]**

b) Show that the vector field $\overline{F} = \left(8xy + z^4\right)\overline{i} + \left(4x^2 - z\right)\overline{j} + \left(4xz^3 - y\right)\overline{k}$ is irrotational. Find Scalar potential function $\phi$ such that $\overline{F} = \nabla\phi$. **[5]**

c) Using Green's theorem for $\overline{F} = xy\overline{i} + y^2\overline{j}$ over region R enclosed by parabola $y = x^2$ and line $y = x$ in the first quadrant, evaluate $\int_c xy\,dx + y^2 dy$. **[5]**

OR

*Q7)* a) Using Stoke's theorem evaluate $\iint_s \nabla \times \overline{F} \cdot \hat{N}\,ds$ where $\overline{F} = 3y\overline{i} - xz^2\overline{j} + yz^2\overline{k}$ and $s$ is surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$. **[5]**

b) Prove that (any one): **[5]**

i) $\overline{b} \times \left(\nabla\left(\overline{a} \cdot \nabla \log r\right)\right) = \frac{\overline{b} \times \overline{a}}{r^2} - \frac{2(\overline{a} \cdot \overline{r})}{r^4}\left(\overline{b} \times \overline{r}\right)$

ii) $\nabla^4\left(r^2 \log r\right) = \frac{6}{r^2}$

c) Find angle between the tangents to the curve $\overline{r} = t^2\overline{i} + 2t\overline{j} - t^3\overline{k}$ at the points $t = 1$ and $t = -1$. **[5]**

**Q8)** a) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is **[8]**

$$u(x,0) = x, \qquad 0 \le x \le 50,$$
$$= 100 - x, \quad 50 \le x \le 100.$$

Find the temperature $u(x, y)$ at any time.

b) Solve following $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ subject to **[7]**

i) $y(0,t) = 0, \forall t$

ii) $y(l,t) = 0, \forall t$

iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

iv) $y(x,0) = \frac{3a}{2l}x, \qquad 0 \le x \le \frac{2l}{3}$

$$= \frac{3a}{l}(l - x), \quad \frac{2l}{3} \le x \le l$$

OR

**Q9)** a) Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to **[8]**

i) $u = 0$ when $y \to \infty$ for all $x$

ii) $u = 0$ when $x = 0$ for all $y$

iii) $u = 0$ when $x = l$ for all $y$

iv) $u = x(1 - x)$ when $y = 0$ for $0 < x < 1$.

b) The initial temperature along the length of an infinite bar is given by $u(x,0) = 2, \ |x| < 1$; $= 0, \ |x| > 1$. If the temperature $u(x,t)$ satisfies the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t > 0,$ find the temperature at any point of the bar at time $t$. **[7]**

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