Total No. of Questions : 9]

SEAT No. :

P601

[Total No. of Pages : 4

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S.E. (Automobile & Mechanical/Mechanical (Sandwich)/Automation & Robotics/Mechatronics)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (207002) (Semester - IV)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory.*
2) *Solve Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn whenever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.*
6) *Assume suitable data, if necessary.*

Q1) a) If $\phi = x^2 - y^2 - z^2$ then $\nabla\phi$ at point (1, 2, 3) is **[2]**

i) $2\hat{i} - 4\hat{j} - 12\hat{k}$ ii) $2\hat{i} - 4\hat{j} + 12\hat{k}$

iii) $2\hat{i} + 4\hat{j} + 12\hat{k}$ iv) $\hat{i} + \hat{j}$

b) The most general solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ representing metal plate having length x and breadth $y \to \infty$ is **[2]**

i) $(c_1 \cos mx + c_2 \sin mx)(c_3 e^{my} + c_4 e^{-my})$

ii) $(c_1 e^{mx} + c_2 e^{-mx})$

iii) $(c_1 \cosh mx + c_2 \sinh mx)(c_3 \cos my + c_4 \sin my)$

iv) $(c_1 e^{mx} + c_2 e^{-mx})$

c) The standard deviation and arithmetic mean of the distribution are 4.89898 and 17 respectively. Coefficient of variation of distribution is **[2]**

i) 26.12 ii) 28.82

iii) 21.82 iv) 25.82

d) X is normally distributed. The mean of X is 0.7 and standard deviation is 0.05. Then probability of $p(x \geq 0.8)$ is (Given : $z = 2$, A = 0.4772) **[2]**

i) 0.5228 ii) 0.0228

iii) 0.9772 iv) 0.4772

P.T.O.

e) Coefficient of correlation always lies between **[1]**

i) $-1 \le r \le 1$
ii) $0 \le r \le 1$
iii) $-2 \le r \le 2$
iv) $-1 \le r \le 0$

f) If $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then ∇r is **[1]**

i) r
ii) $\frac{\overline{r}}{r}$
iii) $\overline{r}$
iv) 0

Q2) a) Fit a straight line for the following data **[5]**

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

b) Calculate first four moments about the mean of the following distribution. **[5]**

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

c) Find the coefficient of correlation for following data. **[5]**

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	47

OR

Q3) a) The results of measurements of electric resistance R of a copper bar at various temperature are listed below. Find a relation R = $a + bt$ **[5]**

t	19	25	30	36	40	45	50
R	76	77	79	80	82	83	85

b) First four moments of a distribution about the value 2 are 1, 2.5, 5.5 and 16. Find first four moments about the mean, β_1 & β_2. **[5]**

c) Obtain regression lines for the following data. **[5]**

x	2	3	5	7	9	10	12	15
y	2	5	8	10	12	14	15	16

Q4) a) A class has 12 boys and 4 girls. Suppose three students are selected at random from the class. Find the probability that they are all boys. **[5]**

b) Out of 2000 families with 4 children each, how many would you expect to you **[5]**

i) At least one boy
ii) 1 or 2 girls

c) In certain city 4000 tube lights are installed. If the lamps have average life of 1500 burning hours with standard deviation 100 hours. Assuming normal distribution. How many lamps will fail in first 1400 hours. How many lamps will last beyond 1600 hours. (Given : A(1) = 0.3413) **[5]**

OR

Q5) a) A and B are playing a game of alternating tossing a coin, one who gets head first wins the game. Find the probability of B winning the game if A has start. **[5]**

b) A certain factory turning cotter pins knows that 2% of his product is defective. If he sells cotter pins and guarantees that not more than 5 pins will be defective in a box, find the approximate probability that a box will fail to meet guaranteed quality. **[5]**

c) The number of computer science books borrowed from a library during a particular week is given below. **[5]**

Day	Mon	Tue	Wed	Thurs	Fri	Sat
Number of book borrowed	140	132	160	148	134	150

Test the hypothesis that the number of books borrowed does not depend on the day. Taking 5% of level of significance $\chi^2_{5,0.05} = 11.07$.

Q6) a) Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at (1, 1, 1) along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. **[5]**

b) Show that the vector field $\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$. **[5]**

c) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ z = 0$ under the force field. **[5]**

$$\bar{F} = (2x - y + z)\bar{i} + (x + y - z^2)\bar{j} + (3x - 2y + 4z)\bar{k}.$$

OR

Q7) a) Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). **[5]**

b) Prove that (any one). **[5]**

i) $\nabla^2\left[\nabla \cdot \left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}$

ii) $\nabla \times \left[\bar{a} \times (\bar{b} \times \bar{r})\right] = \bar{a} \times \bar{b}$

c) Evaluate $\iint_s (\nabla \times \bar{F}) \cdot d\bar{s}$ where $\bar{F} = (x^3 - y^3)\bar{i} - xyz\bar{j} + y^3\bar{k}$ and s is surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. **[5]**

Q8) a) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions: **[8]**

i) u is finite for all t

ii) $u(0, t) = 0 \ \forall t$

iii) $u(l, t) = 0 \ \forall t$

iv) $u(x, 0) = u_0$ (constant) for $0 \le x \le l$, where l is the length of the bar.

b) A string is stretched and fastened to two points L apart. Motion is started by displaying the string in the form $u = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. **[7]**

OR

Q9) a) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = \text{L}$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = \text{L}$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature distribution $u(x, y)$. **[8]**

b) Use Fourier sine transform to solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$, subject to **[7]**

i) $u(0,t) = 0, \ \forall t$

ii) $u(x,0) = e^{-x}, \ x > 0$

iii) u and $\frac{\partial u}{\partial x} \to 0$ as $n \to \infty$

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