Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

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S.E. (Mech./Prod./Auto.) (I Sem.) EXAMINATION, 2019

(Common to Mech. S/W)

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following differential equations : [8]

(*i*) $$\frac{d^2y}{dx^2} + 16y = x\sin 3x + 2^{2x} + 16$$

(*ii*) $$(2x+1)^2\frac{d^2y}{dx^2} - 4(2x+1)\frac{dy}{dx} + 8y = \cos\left[\sqrt{2}\log(2x+1)\right]$$

(*iii*) $$\frac{d^2y}{dx^2} + 25y = \cot 5x,$$

by using the method of variation of parameters.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x)\sin\lambda x\, dx = \begin{cases} 4-\lambda, & 0 \le \lambda \le 4 \\ 0 , & \lambda > 0 \end{cases}$$

P.T.O.

Or

2. (*a*) The differential equation : [4]

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + n^2x = 0,$$

where k is constant and $k < n$ represents the damped harmonic oscillations of a particle. Solve the differential equation and show that the ratio of amplitude of any oscillation to its preceeding one is constant.

(*b*) Solve any *one* of the following : [4]

(*i*) $L\left[e^{-2t} t \sin 2t\right]$

(*ii*) $L^{-1}\left[\frac{s+3}{(s+4)(s-8)}\right]$.

(*c*) Solve the following differential equation by using Laplace transform method : [4]

$$\frac{dx}{dt} + 4x = te^{-4t}$$

where $x(0) = 1$.

3. (*a*) The first four moments of a distribution about 2 are 1, 2.5, 5.5 and 16. Calculate first four moments about the mean. Also obtain the coefficient of skewness (β_1) and coefficient of kurtosis (β_2). [4]

(*b*) In a town, 10 accidents took place in a span of 50 days. Assuming that the number of accidents per day follows Poisson distribution, find the probability that there will be : [4]

(*i*) at least 3 accidents in a day

(*ii*) at least 2 accidents in a day.

(*c*) Find the directional derivative of the function $\phi = x^2y + xyz + z^3$ at $(1, 2, -1)$ along normal to the surface $x^2y^3 = 4xy + y^2z$ at $(1, 2, 0)$. [4]

Or

4. (*a*) Calculate the coefficient of correlation for the following data : [4]

x	y
1	9
2	8
3	10
4	12
5	11
6	13
7	14
8	16
9	15

(*b*) Prove the following (any *one*) : [4]

(*i*) $\overline{F} = \nabla\phi \times \nabla\psi$ is a solenoidal field

(*ii*) $\nabla^4 r^4 = 120$.

(*c*) Show that the vector field : [4]

$$\overline{F} = (y\sin z - \sin x)\overline{i} + (x\sin z + 2yz)\overline{j} + \left(xy\cos z + y^2\right)\overline{k}$$

is irrotational. Also find the scalar potential ϕ such that $\overline{F} = \nabla\phi$.

5. (a) Find the work done in moving a particle in the field [5]

$$\vec{F} = yzi + xzj + zyk$$

along the curve

$$x = 2(t + \sin t), \quad y = 2(1 - \cos t), \quad z = 2t$$

in xy-plane from $t = -\pi$ to $t = \pi$.

(b) Use divergence theorem for [4]

$$\vec{F} = 2xzi + yzj + z^2k$$

over upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

(c) By using Stokes' theorem, evaluate : [4]

$$\int_C \left[e^x dx + 2y dy - dz\right]$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$.

Or

6. (a) Using Green's theorem evaluate : [5]

$$\int_C \left[\left(x^2 + 2y\right)dx + \left(4x + y^2\right)dy\right]$$

where C is the boundary of the region bounded by $y = 0$, $y = 2x$, and $x + y = 3$.

(b) By using Gauss divergence theorem, evaluate [4]

$$\iint_S \vec{r} \cdot \hat{n}\, ds$$

over the closed surface of the sphere of unit radius.

(c) By using Stokes' theorem evaluate : [4]

$$\int_C \left[\left(x^2 - y^2\right) dx + 2xy\, dy\right]$$

in the rectangular region in the xy-plane bounded by the lines $x = -a$, $x = a$, $y = 0$, $y = b$.

7. (a) Solve the equation : [7]

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2},$$

representing the vibration of a string of length l, fixed at both ends, given boundary conditions :

(i) $y(0, t) = 0$,

(ii) $y(l, t) = 0$,

(iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(iv) $y(x, 0) = x$, $0 < x < l$.

(b) Solve : [6]

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

if

(i) $u(0, t) = 0$,

(ii) $u(l, t) = 0$,

(iii) $u(x, t)$ is bounded,

(iv) $u(x, 0) = u_0$.

Or

8. (*a*) Solve : [6]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

which satisfies the conditions :

$u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi x}{l}$.

(*b*) Use Fourier transform to solve the equation : [7]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \ t > 0,$$

subject to conditions :

(*i*) $u(0, t) = 0, \ t > 0$

(*ii*) $u(x, 0) = \begin{cases} 4 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded.