Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[5152]-511

S.E. (Mechanical/Sandwich/Auto.) (I Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Three Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

(*v*) *All* questions are compulsory.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 + 13D + 36)y = e^{-4x} + \sinh x$

(*ii*) $(D^2 - 2D + 2)y = e^x + \tan x$

(using method of variation of parameter)

(*iii*) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x^5.$

(*b*) Using Fourier integral representation show that : [4]

$$\int_0^\infty \frac{\lambda^3 \sin \lambda x}{\lambda^4 + 4}\, d\lambda = \frac{\pi}{2}\, e^{-x} \cos x,\ x > 0.$$

P.T.O.

Or

2. (*a*) A body of weight 9.8 N is suspended from a spring having constant 4 N/m. Prove that the motion is one of the resonance if a force 16 sin $2t$ is applied and damping force is negligible. Assume that initially the weight is at rest in the equilibrium position. [4]

(*b*) Solve any *one* : [4]

(*i*) Find the Laplace transform of :

$$\cosh t \int_0^t e^t \cosh(t)\, dt.$$

(*ii*) Find the Inverse Laplace Transform of $\cot^{-1}\left(\frac{s-2}{3}\right)$.

(*c*) Using Laplace transform solve the D.E. : [4]

$$y'' + 2y' + y = te^{-t},\ y(0) = 1,\ y'(0) = -2.$$

3. (*a*) If [4]

$$\Sigma f = 27,\ \Sigma fx = 91,\ \Sigma fx^2 = 359,$$

$$\Sigma fx^3 = 1567,\ \Sigma fx^4 = 7343.$$

Find the first four moments about origin. Also find μ_2, μ_3, μ_4.

(*b*) An unbiased coin is thrown 10 times. Find the probability of getting exactly 6 heads and at least 6 heads using binomial distribution. [4]

(*c*) Find the directional derivative of $xy^2 + yz^3$ at (2, –1, 1) along the line $2(x - 2) = y + 1 = z - 1$. [4]

Or

4. (*a*) Obtain regression lines for the following data : [4]

x	***y***
6	9
2	11
10	5
4	8
8	7

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

(*ii*) $\nabla \cdot \left[r \, \nabla \left(\frac{1}{r^3} \right) \right] = \frac{3}{r^4}$.

(*c*) Show that the vector field [4]

$$\overline{F} = (x+2y+4z)\overline{i} + (2x-3y-z)\overline{j} + (4x-y+2z)\overline{k}$$

is irrotational and hence find scalar function ϕ such that $\overline{F} = \nabla\phi$.

5. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = x^2\overline{i} + xy\overline{j}$$

and C is the straight line $y = x$, joining (0, 0) and (1, 1).

(*b*) Prove that : [4]

$$\iint_S (\phi\nabla\psi - \psi\nabla\phi)\cdot d\overline{S} \iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV.$$

(*c*) Use Stokes' theorem to evaluate : [5]

$$\int_C \left(4y\overline{i} + 2z\overline{j} + 6y\overline{k}\right)\cdot d\overline{r}$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$.

Or

6. (*a*) Evaluate : [4]

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = xy^2\overline{i} + y\overline{j}$$

and C is curve $x = t$, $y = t^2$, joining $t = 0$ and $t = 1$.

(b) Evaluate : [5]

$$\iint_S \bar{F} \cdot d\bar{s}$$

where

$$\bar{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$$

and S is the upper part of the sphere

$$x^2 + y^2 + z^2 = 1$$

above *xoy* plane.

(c) Evaluate : [4]

$$\iint_S \left(\nabla \times \bar{F}\right) \cdot \hat{n}\, ds$$

where

$$\bar{F} = xy^2\bar{i} + y\bar{j} + z^2x\bar{k}$$

and S is the surface of a rectangular lamina bounded by :

$$x = 0,\ y = 0,\ x = 1,\ y = 2,\ z = 0.$$

7. (a) Solve the wave equation [7]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with boundary conditions :

(i) $u(0, t) = 0, \ \forall t$

(*ii*) $u(l, t) = 0, \forall t$

(*iii*) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0,$

(*iv*) $u(x, 0) = a \sin\frac{\pi x}{l}$

(*b*) Solve the heat equation [6]

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

for the function $u(x, t)$, subject to the following conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0, \forall t$

(*iii*) $u(x, 0) = x, 0 \leq x < l$

(*iv*) $u(x, \infty)$ is finite.

Or

8. (*a*) Solve the Laplace equation [6]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

subject to condition :

(*i*) $u(x, 0) = 0$

(*ii*) $u(x, l) = 0$

(*iii*) $u(\infty, y) = 0,$

(*iv*) $u(0, y) = a_0.$

(*b*) Use Fourier transform to solve : [7]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \ t > 0$$

where $u(x, t)$ satisties the conditions :

(*i*) $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0, \ t > 0$

(*ii*) $u(x, 0) = \begin{cases} x & 0 < x < 1 \\ 0 & x > \end{cases}$

(*iii*) $|u(x, t)| < m.$