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S.E. (Computer/IT Engineering) (Second Semester)

EXAMINATION, 2017

ENGINEERING MATHEMATICS III

(2015 Course)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Neat diagrams must be drawn wherever necessary.

(ii) Figures to the right indicate full marks.

(iii) Your answers will be valued as a whole.

(iv) Use of electronic pocket calculator is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve any *two* : [8]

(i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^{ex}$

(ii) $(D^2 + 4D + 4)y = x^{-3} e^{-2x}$

(iii) $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$

(b) Find the Fourier transform of : [4]

$$f(x) = 1, \quad |x| \leq 1$$

$$= 0, \quad |x| > 1$$

and evaluate $\int_0^\infty \frac{\lambda \cos \lambda x}{\lambda} d\lambda$.

P.T.O.

Or

2. (a) An inductor of 0.5 henries is connected in series with a resistor of 6 ohms, a capacitor of 0.02 farads, a generator having alternative voltage given by $24 \sin 10 t$, $t > 0$ and switch k . Set up a differential equation for this circuit and find charge at time t . [4]

- (b) Solve any one of the following : [4]

(i) Find $z\{f(k)\}$, where $f(k) = 3^k, k < 0$
 $= 2^k, k \geq 0$

- (ii) Find :

$$z^{-1} \left\{ \frac{z^2}{z^2 + 1} \right\}$$

by using inversion integral method.

- (c) Solve the following difference equation : [4]

$$y(k + 2) - 5y(k + 1) + 6y(k) = 36$$

$$y(0) = y(1) = 0.$$

3. (a) Calculate the first four central moments from the following data and hence find β_1 and β_2 : [4]

x	0	1	2	3	4	5	6
f	5	15	17	25	19	14	5

- (b) Fit a straight line to the following data by least square method : [4]

x	0	5	10	15	20	25
y	12	15	17	22	24	30

(c) The number of breakdowns of a computer in a week is a Poisson variable with $\lambda = np = 0.3$. What is the probability that the computer will operate : [4]

(i) with no breakdown and

(ii) at the most one breakdown in a week.

Or

4. (a) The average test marks in a particular class is 79 and standard deviation is 5. If the marks are normally distributed, how many students in a class of 200, did not receive marks between 75 and 82. Given $z = 0.8$, Area = 0.2881 and $z = 0.6$, Area = 0.2257. [4]

(b) An insurance agent accepts policies of 5 men of identical age and in good health. The probability that a man of this age will be alive 30 years hence is $2/3$. Find the probability that in 30 years : [4]

(i) all five men and

(ii) at least one man will be alive.

(c) The two variables x and y have regression lines : [4]

$$3x + 2y - 26 = 0 \text{ and } 6x + y - 31 = 0$$

Find :

(i) the mean values of x and y and

(ii) correlation coefficient between x and y .

5. (a) Find the directional derivative of a scalar point function $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of a vector $4i + 2j + 4k$. [4]

- (b) Show that the vector field :

$$\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$$

is irrotational and hence find a scalar potential function ϕ such that $\bar{F} = \nabla\phi$. [4]

- (c) Find the work done by the vector field : [5]

$$\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$$

in moving a particle of unit mass from $(1, 1, 1)$ to $(2, -1, 2)$.

Or

6. (a) Find the directional derivative of a scalar point function $\phi = xy - z^2 + 2xz$ at $(1, 0, 2)$ in the direction of $4i - j + 2k$. [4]

- (b) Show that (any one) : [4]

$$(i) \quad \nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^n} \right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}, \text{ where } \bar{a} \text{ is a constant vector.}$$

$$(ii) \quad \nabla^2 \left(\nabla \cdot \frac{\bar{r}}{r^2} \right) = \frac{2}{r^4}.$$

- (c) Evaluate the integral $\int_c \bar{F} \cdot d\bar{r}$, along the curve $x = 2t$, $y = t$, $z = 3t$ from $t = 0$ to $t = 1$, where $\bar{F} = 3x^2i + (2xz - y)j + zk$. [5]

7. (a) If :

$$u = -2xy + \frac{y}{x^2 + y^2},$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z . [4]

(b) Evaluate $\oint_c \frac{e^z}{(z+1)(z+2)} dz$, where c is the contour

$$|z + 1| = \frac{1}{2}. \quad [5]$$

(c) Find the Bilinear transformation which maps the point $-i, 0, 2 + i$ of the z -plane onto the points $0, -2i, 4$ of the w -plane. [4]

Or

8. (a) If :

$$u = \frac{1}{2} \log(x^2 + y^2),$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z . [4]

(b) Evaluate $\oint_c \frac{\sin \pi z^2 + 2z}{(z-1)(z-2)} dz$, where c is the circle $|z| = 4$. [5]

(c) Find the image of the circle $(x - 3)^2 + y^2 = 2$ under the transformation $w = \frac{1}{z}$. [4]