

Total No. of Questions : 8]

SEAT No. :

PE4295

[Total No. of Pages : 4

[6582]-68

S.E. (Information Technology, AIML)

DISCRETE MATHEMATICS

(2019 Pattern) (Semester - III) (218541)

Time : 2½ Hours]

[Max. Marks : 70

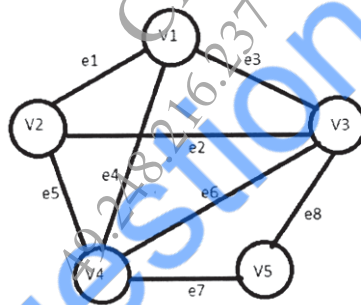
Instructions to the candidates:

- 1) Answer Q1 or Q2, Q3 or Q4, Q5 or Q6, Q7 or Q8.
- 2) Neat diagrams must be drawn wherever necessary.
- 3) Figures to the right indicate full marks.
- 4) Your answers will be valued as a whole.
- 5) Assume suitable data, if necessary.

Q1) a) What is Isomorphism in the graph? Explain the rules for graph to be isomorphic with an example. [5]

b) Construct an optimal tree for the weights 5,7,10,15,35,40. Find the weight of the optimal tree. [6]

c) Find the fundamental cutset and fundamental circuit for the spanning graph with vertices v_4, v_2, v_1, v_3, v_5 [6]



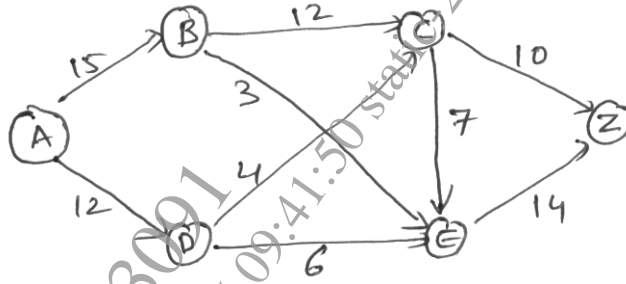
OR

Q2) a) Explain chromatic numbers with respect to the following graphs [5]

- i) Complete Graph
- ii) Star Graph
- iii) Wheel Graph

P.T.O.

- b) Find the maximum flow in the given network [6]



- c) What number of edges are present in a complete graph with n vertices? Explain with the help of a handshaking lemma. [6]

- Q3) a) Consider the following relations on $\{1, 2, 3, 4\}$: [6]

$$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\},$$

$$R_2 = \{(1, 1), (1, 2), (2, 1)\},$$

$$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\},$$

$$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\},$$

$$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\},$$

$$R_6 = \{(3, 4)\}.$$

Which of these relations are reflexive?

- b) Use Warshall's algorithm to find the transitive closure R^* of the following relation R on the set $A = \{1, 2, 3, 4\}$, where [6]

$$R = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (3, 4), (3, 2), (4, 2), (4, 3)\}$$

- c) Show that if seven numbers from 1 to 12 are chosen then two of them will add up to 13. [6]

OR

- Q4) a) Consider these relations on the set of integers : [6]

$$R_1 = \{(a, b) \mid a \leq b\},$$

$$R_2 = \{(a, b) \mid a > b\},$$

$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a, b) \mid a = b\},$$

$$R_5 = \{(a, b) \mid a = b + 1\},$$

$$R_6 = \{(a, b) \mid a + b \leq 3\}$$

Which of these relations contain each of the pairs $(1, 1)$, $(1, 2)$, $(2, 1)$, $(1, -1)$, and $(2, 2)$?

- b) Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$. Which ordered pairs are in the relation R and R^{-1} represented by the matrix? [6]

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- c) Show that 7 colors are used to paint 50 bicycles, then at least 8 bicycles of the same color [6]

- Q5) a) Determine quotient and remainder for the following : [6]

i) $88 \div 10$

ii) $88 / 10$

- b) State and explain the Euclidean algorithm to compute GCD of two numbers. [4]

- c) Let $a = 37$ and $b = 249$. Find [8]

i) $d = \gcd(a, b)$

ii) Find integers m and n such that $d = ma + nb$

OR

- Q6) a) Find the multiplicative inverse of 40 mod 197 using the Extended Euclidean algorithm. [6]

- b) Using the primality test theorem determine if the following numbers are prime. Justify your answer : [4]

i) 131

ii) 253

- c) Using the Chinese Remainder Theorem find the value of X such that : [8]

$$X \equiv 2 \pmod{3}$$

$$X \equiv 4 \pmod{5}$$

$$X \equiv 3 \pmod{7}$$

Q7) a) Let $R = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\}$ and $*$ = binary operation, so that $a*b$ is overall angular rotation corresponding to successive rotations by a and then by b . Show that $(R, *)$ is a group. [6]

b) Consider the (2, 6) encoding function e . $e(00) = 10111100$, $e(10) = 10101010$, $e(01) = 001111010$, $e(11) = 10101101$

Find the minimum distance of e . How many errors will e detect? [8]

c) Define Ring with Unity, with an example. [3]

OR

Q8) a) Prove that the following table on relation of elements of set $G = \{0,1,2,3,4,5\}$ multiplication mod 6 is not a group. [6]

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

b) $S = \{1,2,3,6,12,28\}$, where $a*b$ is defined as $GCD(a,b)$. Determine whether it is an Abelian Group or not. [8]

c) Define Commutative Ring, with an example. [3]

x x x