Total No. of Questions : 8]

SEAT No. :

P1596

[Total No. of Pages : 5

[6002]-226

S.E. (Information Technology) (Artificial Intelligence & Machine Learning)

DISCRETE MATHEMATICS

(214441, 218541) (2019 Pattern) (Semester - III)

Time : 2½ Hours] *[Max. Marks : 70*

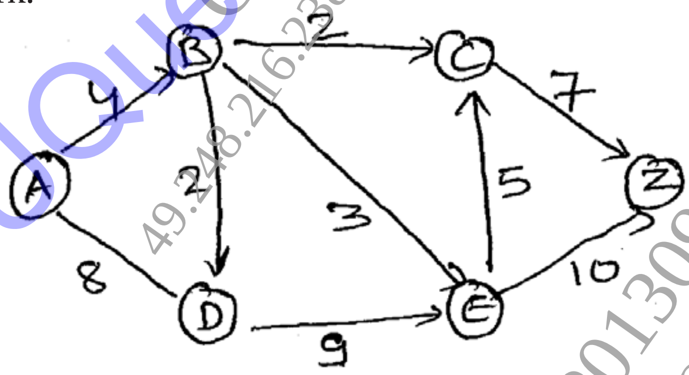
Instructions to the candidates:

1) *Solve Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate fill marks.*
4) *Assume suitable data, if necessary.*

Q1) a) Show that the maximum number of edges in a simple graph with n vertices is n.(n-1)/2. **[5]**

b) Construct an optimal tree for the weights 3,5, 9, 18, 30, 40, 55. Find the weight of the optimal tree. **[6]**

c) Using the labelling procedure, find the max flow for the following transport network. **[6]**

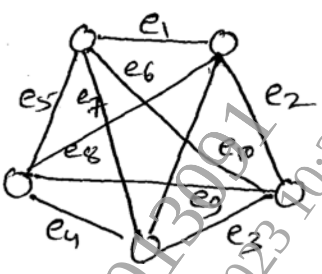


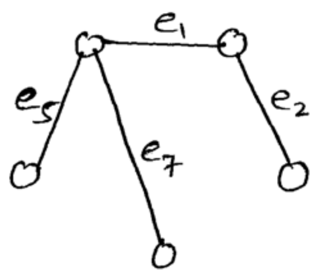
OR

Q2) a) Determine the number of edges in a graph with 7 nodes, 2 of degree 4, 2 of degree 3 and 3 of degree 2. Draw one such graph. **[5]**

P.T.O.

b) Find the fundamental system of cutsets and fundamental system of the circuit for graph, G with respect to the spanning tree, T. **[6]**





c) Find the chromatic number with the help of graph coloring for: **[6]**

i) K6 (complete graph with 6 vertices)

ii) Any complete bipartite graph.

iii) C7 (cyclic graph with 7 vertices).

Q3) a) Consider these relations on the set of integers : **[6]**

R1 = $\{(a, b) \mid a \leq b\}$;

R2 = $\{(a, b) \mid a > b\}$;

R3 = $\{(a, b) \mid a = b \text{ or } a = -b\}$;

R4 = $\{(a, b) \mid a = b\}$;

R5 = $\{(a, b) \mid a = b + 1\}$;

R6 = $\{(a, b) \mid a + b \leq 3\}$;

Which are symmetric and which are antisymmetric?

b) Functions, *f*, *g* & *h* are defined on the set X = {1, 2, 3} as **[6]**

f = {(1, 3), (2, 1), (3, 2)}

g = {(1, 2), (2.3), (3, 1)}

h = {(1, 2), (2, 1), (3, 3)}

i) Find *fog* and gofs Are they equals?

ii) Find *fogoh* and *fohog*.

c) If A = {a,b,c,d} and R = {(a,b),(c,d),(c,c),(d,a),(a,a),(b,b),(d,d)}is a relation on A. Draw a digraph R and $\bar{R}$. **[6]**

OR

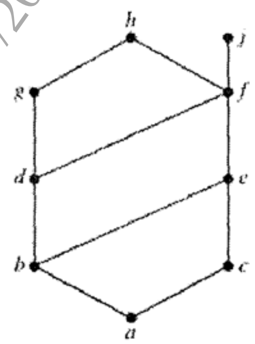
Q4) a) Let A = B be the set of real numbers. **[6]**

$f : a \to$ given by $f(x) = 2x^3 - 1$

$g : B \to A$ given by $g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$

Show that f is a bijection between A and B and g is a bijection between B and A.

b) **[6]**



i) Find the lower and upper bounds of the subsets {a, b, c}, {j, h}, and {a, c, d, f } in the post with the Hasse diagram shown in Figure?

ii) Find the greatest lower bound and the least upper bound of {b, d, g}, if they exist, in the post shown in Figure?

c) Solve the following recurrence relation **[6]**

$a_r - 3a_{r-1} = 2 \quad , \quad r >= 1, a_0 = 1$

Q5) a) Using Euclidean Algorithm find GCD of 268 & 884. **[6]**

b) Using Fermat's Theorem and Fermat's Euler Theorem solve the following: **[6]**

i) 7^121 mod 4

ii) 11^100 mod 17

c) Find the multiplicative Inverse of 37 mod 26 using Extended Euclidean Algorithm. **[6]**

OR

Q6) a) Using the Chinese Remainder Theorem, find the value of P using the following data. **[8]**

P = 1 mod 2

P = 2 mod 3

P = 3 mod 5

b) State and explain Fermat - Euler's Theorem with example. **[4]**

c) Find the Totient function of the following numbers : **[6]**

i) 75

ii) 143

iii) 108

Q7) a) Let G = {even, odd) and binary operation $\oplus$ be define as, **[6]**

$\oplus$	even	odd
even	even	odd
odd	odd	even

Show that (G, $\oplus$) is a group

b) Define the following terms with an example : **[6]**

i) Monoid

ii) Group

iii) Abelian group

iv) Ring

c) Find the hamming distance between code words of: C = {(0000), (0101), (1011), (0111), (1111)}

Rewrite the message by adding an even parity check bit and odd parity check bit. **[5]**

OR

Q8) a) Consider the (2,6) encoding function e. e(00)=100000, e(10)=101010 e(01)=001 110, e(11)=101001 **[6]**

i) Find the minimum distance of e

ii) How many errors will e detect?

b) Let I be the set of all integers. For each of the following determine whether * is an associative operation or not : **[8]**

i) a*b = max(a,b)

ii) a*b = min(a + 2,b)

iii) a*b = 2a - 2b

iv) a*b = min(2a - b, 2b - a)

v) a*b = LCM(a,b)

vi) a*b = a/b

vii) a*b = power(a,b)

viii) $a*b = a^2 + 2b + ab$

c) Define field with an example. **[3]**

x x x