

Total No. of Questions : 9]

SEAT No. :

PB-3594

[Total No. of Pages : 4

[6260]-9

F.E. (All Branches)

ENGINEERING MATHEMATICS - II
(2019 Pattern) (Semester - I/II) (107008)

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) Q.No.1 is compulsory.
- 2) Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.No.9.
- 3) Neat diagram must be drawn wherever necessary.
- 4) Figures to the right indicate full marks.
- 5) Use of electronic pocket calculator is allowed.
- 6) Assume suitable data, if necessary.

Q1) Write the correct option for the following multiple choice equations.

a) $\int_0^{\infty} e^{-x} x^{1/2} dx$ is equal to [2]

i) $\frac{1}{2}$

ii) $\frac{\sqrt{\pi}}{2}$

iii) $\frac{\pi}{2}$

iv) $\sqrt{\pi}$

b) $\int_0^{\pi/2} \cos^4 \theta d\theta$ is equal to [2]

i) $\frac{3\pi}{16}$

ii) $\frac{3\pi}{4}$

iii) 0

iv) $\frac{3\pi}{8}$

c) The region of absence for the curve represented by the equation $y^2(2a - x) = x^3$ is [2]

i) $x > 0, x < 2a$

ii) $x < 0, x > 2a$

iii) $x < 0, x < 2a$

iv) $x > 0, x > 2a$

P.T.O.

- d) The centre and radius of sphere $x^2 + y^2 + z^2 - 2z - 3 = 0$ is [2]
- i) (0, 0, 1) and 2 ii) (0, 0, 0) and 3
- iii) (1, 0, 0) and 2 iv) (0, 0, -2) and 3

- e) The value of $\int_0^1 \int_0^x dx dy$ is [1]
- i) $\frac{1}{3}$ ii) $\frac{1}{3}x$
- iii) $\frac{1}{2}$ iv) $\frac{1}{3}y$

- f) $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{5}{2}} dx$ is equal to [1]
- i) $\beta(\frac{5}{2}, \frac{7}{2})$ ii) $\beta(-\frac{1}{2}, \frac{1}{2})$
- iii) $\frac{1}{2}\beta(\frac{5}{2}, \frac{3}{2})$ iv) $\beta(\frac{1}{2}, \frac{5}{2})$

Q2) a) If $\text{In} = \int_{\pi/4}^{\pi/2} \cot^n \theta d\theta$, then prove that $\text{In} = \frac{1}{n-1} - \text{In} - 2$. [5]

b) Evaluate $\int_0^{\infty} x^8 e^{-2x^2} dx$ [5]

c) Show that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} [\text{erf}(b) - \text{erf}(a)]$. [5]

OR

Q3) a) If $\text{In} = \int_0^{\pi/2} x^n \cos x dx$, then show that $\text{In} = \left(\frac{\pi}{2}\right)^n - n(n-1)\text{In} - 2$. [5]

b) Show that $\int_0^1 x^{m-1}(1-x^2)^{n-1} dx = \frac{1}{2} \beta\left(\frac{m}{2}, \frac{1}{2}n\right)$ [5]

c) Evaluate $\int_0^1 \frac{x^a - x^b}{\log x} dx$ $0 < a < 1, 0 < b < 1$. [5]

Q4) a) Trace the curve $y^2(a - x) = x^3$. [5]

b) Trace the curve $r = \frac{a}{2}(1 + \cos \theta)$. [5]

c) Find the length of the upper arc of one loop of lemniscate $r^2 = a^2 \cos 2\theta$. [5]

OR

Q5) a) Trace the curve $xy^2 = a^2(a - x)$. [5]

b) Trace the curve $r = a \sin 2\theta$. [5]

c) Trace the curve $x = a(t - \sin t), y = a(1 - \cos t)$. [5]

Q6) a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. [5]

b) Find the equation of right circular cone whose vertex is at $(1, 2, -3)$, semi-vertical angle $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ & axis is the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+1}{-1}$. [5]

c) Find the equation of right circular cylinder of radius 5 and axis is $\frac{x-2}{2} = \frac{y-3}{1} = \frac{z+1}{1}$. [5]

OR

Q7) a) Prove that the two spheres $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$ and $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other and find the point of contact. [5]

b) Find the equation of right circular cone which passes through the point $(1, 1, 2)$ and has the axis as the line $6x = -3y = 4z$ and vertex at origin. [5]

c) Find the equation of right circular cylinder of radius 2, whose axis passes through $(1, 2, 3)$ and has direction cosines proportional to 2, 1, 2. [5]

Q8) a) Evaluate $\iint_R x^2 y^2 dy dx$ over positive quadrant of $x^2 + y^2 = 1$. [5]

b) Find the area between the curve $y = x^2 - 2x - 8$ and x -axis. [5]

c) Find the position of the centroid of the area bounded by the curve $y^2(2a - x) = x^3$ and its asymptote. [5]

OR

Q9) a) Change the order of integration $\int_0^2 \int_0^{2-x} y dy dx$ and then solve it. [5]

b) Evaluate $\iiint_V \frac{dx dy dz}{\sqrt{1 - x^2 - y^2 - z^2}}$ taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$ in the positive octant. [5]

c) Find the moment of inertia (M.I.) of the area enclosed by $r = a(1 - \cos\theta)$ about the line $\theta = \frac{\pi}{2}$. [5]

