

Total No. of Questions : 9]

SEAT No. :

P3924

[6001]-4009

[Total No. of Pages : 4

F.E.

ENGINEERING MATHEMATICS-II
(2019 Pattern) (Semester - I/II) (107008)

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *Q. No.1 is compulsory.*
- 2) *Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
- 3) *Neat diagrams must be drawn wherever necessary.*
- 4) *Figures to the right indicate full marks.*
- 5) *Use of electronic pocket calculator is allowed.*
- 6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions

a) $\int_0^{\frac{\pi}{2}} \sin^4 t \, dt =$ [2]

i) $\frac{3\pi}{16}$

ii) $\frac{3}{8}$

iii) $\frac{3}{16}$

iv) $\frac{3\pi}{8}$

b) The equation of the tangent to the curve $y(1+x^2) = x$ at origin, if exist is [2]

i) $X=0$

ii) $Y=0$

iii) $x = 1, x = -1$

iv) $y = x$

c) The value of double integration $\int_0^1 \int_0^1 \frac{1}{1+x^2} \cdot \frac{1}{1+y^2} \, dx \, dy =$ [2]

i) $\frac{\pi}{2}$

ii) $\frac{\pi^2}{2}$

iii) $\frac{\pi}{4}$

iv) $\frac{\pi^2}{8}$

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d) Centre (C) of sphere $x^2 + y^2 + z^2 - 2z = 4$ is [2]

i) $C \equiv (0, 0, 0)$ ii) $C \equiv (0, 0, 1)$

iii) $C \equiv (0, 1, 0)$ iv) $C \equiv (1, 0, 0)$

e) The curve $r = 2a \sin \theta$ is symmetrical about [1]

i) Pole ii) $\theta = 0$

iii) $\theta = \frac{\pi}{2}$ iv) $\theta = \frac{\pi}{4}$

f) $\iiint_v dx dy dz$ represents [1]

i) Area ii) Mass

iii) Mean Value iv) Volume

Q2) a) If $I_n = \int_0^{\frac{\pi}{4}} \sec^n \theta d\theta$, then prove that $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$ [5]

b) Evaluate $\int_2^5 (x-2)^3 (5-x)^2 dx$ [5]

c) Using DUIS, prove that $\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log \left(\frac{a^2 + 1}{2} \right), a > 0$ [5]

OR

Q3) a) Evaluate

i) $\int_0^{2\pi} \sin^2 \frac{\theta}{2} \cos^{10} \frac{\theta}{2} d\theta$ [3]

ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 t dt$ [2]

b) Evaluate : $\int_0^1 (x \log x)^4 dx$ [5]

c) Prove that: $\frac{1}{x} \frac{d}{da} \operatorname{erfc}(ax) = -\frac{1}{a} \frac{d}{dx} \operatorname{erf}(ax)$ [5]

Q4) a) Trace the curve $x^2 y^2 = a^2 (y^2 - x^2)$. [5]

b) Trace the curve $r = a(1 + \cos \theta)$. [5]

c) Find the are length of Astroid $x^{2/3} + y^{2/3} = a^{2/3}$ [5]

OR

Q5) a) Trace the curve $x^3 + y^3 = 3axy$. [5]

b) Trace the curve $r = a \cos 2\theta$ [5]

c) Trace the curve $x = a(t + \sin t)$, $y = a(1 + \cos t)$. [5]

Q6) a) Show that the plane $x - 2y - 2z = 7$ touches the sphere $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$. Also find the point of contact. [5]

b) Find the equation of right circular cone whose vertex is at origin, whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has a semi-vertical angle of 60° . [5]

c) Find the equation of right circular cylinder of radius 3 and axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. [5]

OR

Q7) a) Show that the two spheres: $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$ touches externally. Also find the point of contact. [5]

b) Find the equation of right circular cone whose vertex is at $(0,0,10)$, axis is the Z-axis and the semi-vertical angle is $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ [5]

c) Find the equation of right circular cylinder of radius $\sqrt{6}$, whose axis passes through the origin and has direction cosines $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$. [5]

Q8) a) Evaluate $\iint_R xy \, dx \, dy$, where R is $x^2 = y, y^2 = -x$. [5]

b) Find area of cardioide $r = a(1 + \cos \theta)$ using double integration. [5]

c) Find the moment of inertia of one loop of the lemniscate $r^2 = a^2 \cos 2\theta$ about initial line. Given that density $\rho = \frac{2m}{a^2}$, m is a mass of the area. [5]

OR

Q9) a) Change order of integration $\int_0^5 \int_{2-x}^{2+x} f(x, y) \, dx \, dy$. [5]

b) Find the volume bounded by the cone $x^2 + y^2 = z^2$ and paraboloid $x^2 + y^2 = z$. [5]

c) Find the x - co-ordinate of centre of gravity of one loop of $r = a \cos 2\theta$, which is in the first quadrant, given that area of loop is $A = \frac{\pi a^2}{8}$. [5]