Total No. of Questions : 11]

SEAT No. :

PC-5147

[Total No. of Pages : 5

[6351]-111
F.E.
BSC - 101-BES : ENGINEERING MATHEMATICS - I
(2024 Pattern) (Semester - I)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Q. 1 is Compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9, Q.10 or Q.11.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

i) $\lim_{x \to 0} \frac{1-\cos x}{x}$ is equal to ________ **[2]**

a) 0 b) 1

c) –1 d) 2

ii) if $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to ______ **[2]**

a) 0 b) yx^{y-1}

c) $x^y \log x$ d) None of these

iii) If x = r cosθ, y = r sinθ then the value of $\frac{\partial(x, y)}{\partial(r, \theta)}$ is ________ **[2]**

a) r b) cosθ

c) 1 d) $\frac{1}{r}$

P.T.O.

iv) Rank of matrix $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is ______ **[2]**

a) 4 b) 3

c) 2 d) 1

v) if $A^{-1} = A^{T}$ then, matrix A is ________ **[1]**

a) Orthogonal b) Singular

c) Regular d) None of above

vi) The eigen values of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & -2 & 2 \end{bmatrix}$ are ________ **[1]**

a) –2, 3, –1 b) –1, –2, –3

c) 1, 2, 3 d) 2, –2, 1

Q2) a) Prove that , is $0 < a < b$ $\dfrac{b-a}{1+b^2} < \tan^{-1}(b) - \tan^{-1}(a) < \dfrac{b-a}{1+a^2}$. **[4]**

b) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. **[4]**

c) Find the Fourier series for the function. $f(x) = x^2, -\pi < x < \pi$ and $f(x + 2\pi) = f(x)$. **[4]**

OR

Q3) a) Verify Rolle's theorem for the function $f(x) = x^2, x \in (-1,1)$. **[4]**

b) Expand $\sqrt{1+\sin x}$ upto x^6. **[4]**

c) Obtain the half range sine series for the function $f(x) = 2-x, 0 < x < 2$. **[4]**

Q4) a) if $u = \log(x^2 + y^2)$, verify $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$ **[4]**

b) If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u).$$ **[4]**

c) If $z = f(u,v)$ and $u = \log(x^2 + y^2)$, $v = y/x$ then show that $x\frac{\partial z}{\partial y} - y\frac{\partial z}{\partial x} = (1+v^2)\frac{\partial z}{\partial v}$. **[4]**

OR

Q5) a) If $u = 2x + 3y$, $v = 3x - 2y$ then find the value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial x}{\partial u}\right)_y \left(\frac{\partial v}{\partial y}\right)_u$ **[4]**

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2)\sin^2 u.$$ **[4]**

c) If $u = f(x - y, y - z, z - x)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **[4]**

Q6) a) Examine the functional dependence for $u = \sin^{-1}x + \sin^{-1}y$; **[4]**

$v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$, if dependent, find relation between them.

b) Examine maxima and minima for $f(x,y) = 4x - x^2 - y^2$, also find the extreme value of the function. **[4]**

c) If $u^3 + v^3 + x + y = 0$ and $x^3 + y^3 + u + v = 0$ then find $\frac{\partial u}{\partial y}$. **[4]**

OR

Q7) a) If $x + y + u^2 - v^2 = 0$ and $u + v + x^2 - y^2 = 0$ find $\frac{\partial(u,v)}{\partial(x,y)}$ **[4]**

b) While calculating the volume of right circular cone, the errors of 2% and 1% are made in measuring the height and radius of the base of right circular cone respectively find the percentage error in calculated volume. **[4]**

c) Find the minimum distance from origin to the plane $3x + 2y + z = 12$, using Lagrange's method of undetermined multiplier method. **[4]**

Q8) a) Examine consistency for the system of equations, $x+y-3z=1$; $4x-2y+6z=8$; $15x-3y+8z=20$. **[4]**

b) Examine for linear dependence and find relation if following vectors are dependent (1, 1, 3), (1, 2, 4) & (1, 0, 2). **[4]**

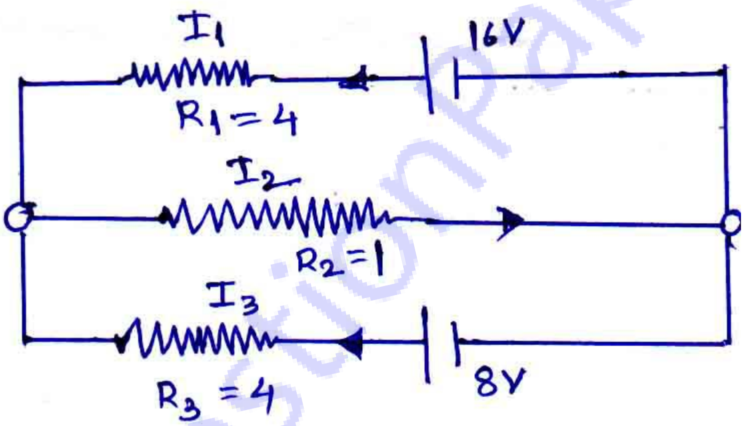
c) Show that $A=\frac{1}{3}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ is orthogonal matrix and hence write A^{-1}. **[4]**

OR

Q9) a) Examine consistency for the following set of equations and obtain solution if consistant $2x-y-z=2$; $x+2y+z=2$; $4x-7y-5z=2$. **[4]**

b) Examine the vectors for linear dependence and if dependent find relation between them, (1, 1, 1), (1, 2, 3), (2, 3, 8) and (3, 4, 9). **[4]**

c) Find the currents I_1, I_2, I_3 in the following circuit as shown in the figure. **[4]**



Q10) a) Verify Cayley Hamilton theorem for $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ Hence find A^{-1}, if it exists. **[6]**

b) Find modal matrix p to reduce $B=\begin{bmatrix}1 & 0 & 0\\ 0 & 3 & -1\\ 0 & -1 & 3\end{bmatrix}$ to diagonal form write the corresponding linear transformation. **[6]**

OR

Q11) a) Find eigen values and corresponding eigen vectors for

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ **[6]**

b) Express $Q(x) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_3$ to canonical form by orthogonal transformation write the orthogonal transformation. **[6]**

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