Total No. of Questions : 9]

SEAT No. :

P9066

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F.E.

ENGINEERING MATHEMATICS - I

(2019 Pattern) (Semester - I/II) (Credit System) (107001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions.

a) If $u = x^3 + y^3 - 3xy$ then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to **[1]**

i) 3 ii) –3

iii) 2 iv) 0

b) If $x = r\cos\theta,\ y = r\sin\theta$ then the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$ is **[1]**

i) $\frac{1}{r}$ ii) r

iii) r^2 iv) None

c) The vectors $X_1 = (-1,0,3),\ X_2 = (2,4,6)$ are **[2]**

i) linearly dependent ii) linearly independent

iii) mutually orthogonal iv) none of these

d) The characteristic equation for the square matrix A is **[2]**

i) $|A - \lambda I| = 0$ ii) $|A + \lambda I| = 0$

iii) $|A^2 - \lambda I| = 0$ iv) None

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e) If $u=\sin^{-1}\frac{\sqrt{x^2+y^2}}{x+y}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to **[2]**

i) u ii) $2u$

iii) 0 iv) None

f) If $x=u(1-v),\ y=uv$ then $\frac{\partial(x,y)}{\partial(u,v)}$ **[2]**

i) u ii) $\frac{1}{u}$

iii) uv iv) $u-uv$

Q2) a) If $u=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$ then show that $\frac{\partial^2 u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$. **[5]**

b) If $f(x,y)=\frac{1}{x^2}+\frac{\ln x-\ln y}{x^2+y^2}$, using Euler's theorem find xf_x+yf_y. **[5]**

c) If $u=f\left(e^{y-z},e^{z-x},e^{x-y}\right)$, find the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$. **[5]**

OR

Q3) a) If $x=u\tan v,\ y=u\sec v$, prove that $\left(\frac{\partial u}{\partial x}\right)_y\cdot\left(\frac{\partial v}{\partial x}\right)_y=\left(\frac{\partial u}{\partial y}\right)_x\cdot\left(\frac{\partial v}{\partial y}\right)_x$. **[5]**

b) If $u=\ln x+\ln y$ find the value of $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}+xu_x+yu_y$. **[5]**

c) If $z=f(u,v)$ and $u=x\cos\theta-y\sin\theta,\ v=x\sin\theta+y\cos\theta$ where θ is a constant, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=u\frac{\partial z}{\partial u}+v\frac{\partial z}{\partial v}$. **[5]**

Q4) a) If $x=u\cos v,\ y=u\sin v$, prove that $JJ'=1$. **[5]**

b) As certain whether the following functions are functionally dependent, if so find the relation between then $u=\frac{x+y}{1-xy},\ v=\tan^{-1}x+\tan^{-1}y$. **[5]**

c) Find the maximum and minimum values of $3x^2-y^2+x^3$. **[5]**

OR

Q5) a) If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **[5]**

b) In calculating volume of right circular cylinder, errors of 2% and 1% are found in measuring height and base radius respectively. Find the percentage error in calculating volume of the cylinder. **[5]**

c) Use Lagrange's method to find the minimum distance from origin to the plane $3x + 2y + z = 12$. **[5]**

Q6) a) Examine following system for consistency $x + y - 3z = 1$; $4x - 2y + 6z = 8$; $15x - 3y + 9z = 20$. **[5]**

b) Examine for linear dependancy or independance of following set of vectors. If dependent, find the relation between them $X_1 \equiv (3,1,1)$, $X_2 \equiv (2,0,-1)$, $X_3 \equiv (1,1,2)$. **[5]**

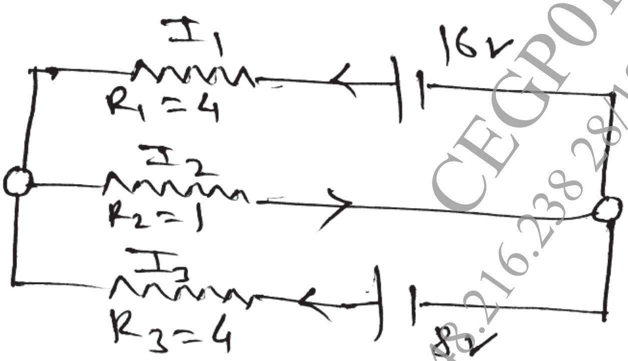
c) Show that $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{bmatrix}$ is orthogonal matrix & hence find A^{-1}. **[5]**

OR

Q7) a) Determine values of k, for which following system have non-trivial solution. $5x + 2y - 3z = 0$; $3x + y + z = 0$; $2x + y + kz = 0$ **[5]**

b) Show that following set of vectors are linearly dependant $X_1 \equiv (2,3,4,-2)$, $X_2 \equiv (-1,-2,-2,1)$, $X_3 \equiv (1,1,2,-1)$ **[5]**

c) Find the currents I_1, I_2, I_3 in the circuit, shown in the figure :- **[5]**



Q8) a) Find eigen values and corresponding eigen vectors of the following matrix

$$A = \begin{bmatrix} 1 & -2 \\ -3 & 0 \end{bmatrix}.$$ **[5]**

b) Verify Cayley Hamilton theorem for given matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **[5]**

c) Find the modal matrix P which diagonalises the given matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **[5]**

OR

Q9) a) Find eigen values and eigen vector corresponding to largest eigen value of a following matrix $A = \begin{bmatrix} 15 & 0 & -15 \\ -3 & 6 & 9 \\ 5 & 0 & -5 \end{bmatrix}$. **[5]**

b) Verify Cayley Hamilton theorem and hence find A^{-1} for given matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}.$$ **[5]**

c) Express the following quadratic form as "sum of the squares form" by consruent transformation. Write down the corresponding linear transformation $Q(x) = x_1^2 + 6x_2^2 + 18x_3^2 + 4x_1x_2 + 8x_1x_3 - 4x_1x_3$. **[5]**

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