Total No. of Questions : 9]

SEAT No. :

P-3926

[Total No. of Pages : 5

[6001]-4001

F.E.

ENGINEERING MATHEMATICS - I

(2019 Pattern) (Semester - I) (107001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory.*
2) *Solve Q. No. 2 or Q. No. 3, Q. No. 4 or Q. No. 5, Q. No. 6 or Q. No. 7, Q. No. 8 or Q. No. 9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions :

a) If $u = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{x^2+y^2}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to **[2]**

i) 2u

ii) –2u

iii) 0

iv) None

b) If $u = x^y$ then $\frac{\partial u}{\partial y}$ is equal to **[1]**

i) 0

ii) yx^{y-1}

iii) $x^y \log x$

iv) x^{y-1}

c) If $x = uv,\ y = \frac{u}{v}$ then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is **[2]**

i) $\frac{-2u}{v}$

ii) uv

iii) $\frac{v}{2u}$

iv) $\frac{-v}{2u}$

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d) A is orthogonal matrix then A^{-1} equal to **[1]**

i) A　　ii) A^T

iii) A^2　　iv) 1

e) For what value of K the homogeneous system $x + 2y - z = 0$, $3x + 8y - 3z = 0$; $2x + 4y + (k-3)z = 0$ has infinitely many solution. **[2]**

i) K = 0　　ii) K = 1

iii) K = 2　　iv) K = 3

f) Using Cayley Hamilton theorem A^{-1} for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ is calculated from **[2]**

i) $\frac{1}{5}(-A - 4I)$　　ii) $\frac{1}{5}(A - 4I)$

iii) $\frac{1}{5}(A + 4I)$　　iv) $\frac{1}{5}(4I - A)$

Q2) a) If $u = ln(x^2 + y^2)$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **[5]**

b) If $e^{2u} = y^2 - x^2$, $\operatorname{cosec} v = \frac{y}{x}$ then find the value of **[5]**

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right).\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$$

c) If $u = f(x - y, y - z, z - x)$ then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$. **[5]**

OR

Q3) a) If $u = ax + by$, $v = bx - ay$ find the value of $\left(\frac{\partial u}{\partial x}\right)_y . \left(\frac{\partial x}{\partial u}\right)_v$. **[5]**

b) If $T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2}$, find the value of $x\frac{\partial T}{\partial x} + y\frac{\partial T}{\partial y}$. **[5]**

c) If $u = f(r, s)$ where $r = x^2 + y^2$, $s = x^2 - y^2$ then show that

$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 4xy\frac{\partial u}{\partial r}.$$ **[5]**

Q4) a) If $x = u + v$, $y = v^2 + w^2$, $z = u^3 + w^3$ then find $\frac{\partial u}{\partial x}$. **[5]**

b) In calculating resistance R of a circuit by using the formula :

$$R = \frac{V}{I}$$

errors of 3% and 1% are made in measuring Voltage V and current I respectively. Find the % error in the calculated resistance. **[5]**

c) Discuss the maxima and minima of : **[5]**

$f(x, y) = x^2 + y^2 + xy + x - 4y + 5$

OR

Q5) a) If $u + v^2 = x$, $v + w^2 = y$, $w + u^2 = z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **[5]**

b) Examine for functional dependence : **[5]**

$u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$

c) A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and it's surface begins to heat. After one hour, the temperature at the point (x, y, z) on the surface of the probe is

$T(x, y, z) = 8x^2 + 4yz - 16z + 600$.

Find the hottest point on the surface of the probe, by using Lagrange's method. **[5]**

Q6) a) Examine for consistency and if consistent then solve it **[5]**

$2x + 3y + 5z = 1$; $3x + y - z = 2$; $x + 4y - 6z = 1$

b) Examine whether the vectors **[5]**

$X_1 = (1, 1, -1, 1)$; $X_2 = (1, -1, 2, -1)$; $X_3 = (3, 1, 0, 1)$

are linearly independent or dependent. If dependent find relation between them.

c) If $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal **[5]**

Find a, b, c.

OR

Q7) a) Investigate for what values of k, the equations **[5]**

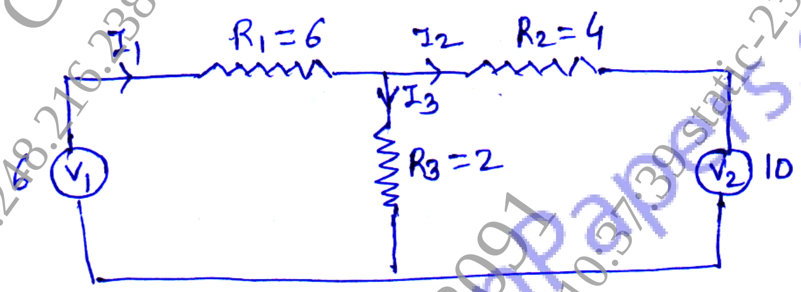
$x + y + z = 1;\ 2x + y + 4z = k;\ 4x + y + 10z = k^2$ have infinite number of solution? Hence find solution.

b) Examine whether the vectors. **[5]**

$X_1 = (2, 3, 4, -2);\ X_2 = (-1, -2, -2, 1);\ X_3 = (1, 1, 2, -1)$

are linearly independent or dependent. If dependent find relation between them.

c) Find the current $I_1;\ I_2;\ I_3$ in the circuit shown in the figure **[5]**



Q8) a) Find eigen values and eigen vectors of the following matrix **[5]**

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and use it to find A^{-1}. **[5]**

c) Find the modal matrix p which transform the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. **[5]**

OR

Q9) a) Find eigen values and eigen vectors of the following matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **[5]**

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and use it to find A^{-1}. **[5]**

c) Reduce the following quadratic form to the "sum of the squares form". **[5]**

$$Q(x) = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz$$

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