Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[5667]-1001

F.E. (I Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—I

(Phase–II)

(2019 PATTERN)

Time : 2½ Hours **Maximum Marks : 70**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Use of electronic pocket calculator is allowed.

(*iii*) Assume suitable data, if necessary.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Figures to the right indicate full marks.

1. (*a*) If $z = \tan(y + ax) + (y - ax)^{3/2}$, find the value of

$$\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}.$$ [6]

(*b*) If $T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2}$, by using Euler's theorem find $x\frac{\partial T}{\partial x} + y\frac{\partial T}{\partial y}$. [6]

(*c*) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$, then show that

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2}\ \frac{\partial z}{\partial u}.$$ [6]

Or

2. (*a*) If $x = u \tan v$, $y = u \sec v$, prove that : [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x .$$

(*b*) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, by using Euler's theorem.

prove that : [6]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u).$$

(*c*) If $x = \frac{\cos\theta}{u}$, $y = \frac{\sin\theta}{u}$ and $z = f(x, y)$, then show that : [6]

$$u\frac{\partial z}{\partial u} - \frac{\partial z}{\partial \theta} = (y - x)\frac{\partial z}{\partial x} - (y + x)\frac{\partial z}{\partial y}.$$

3. (*a*) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = xy + yz + zx$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. [6]

(*b*) Examine whether $u = \frac{x-y}{1+xy}$, $v = \tan^{-1} x - \tan^{-1} y$ are functionally dependent, if so find the relation between them. [5]

(*c*) Find the extreme values of $x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$. [6]

Or

4. (*a*) If $u = x + y^2$, $v = y + z^2$, $w = z + x^2$, using Jacobian find $\frac{\partial x}{\partial u}$. [6]

(*b*) A power dissipated in a resistor is given by $P = \frac{\varepsilon^2}{R}\cdot$ If errors of 3% and 2% are found in ε and R respectively, find the percentage error in P. [5]

(*c*) Using Lagrange's method find extreme value of xyz if $x + y + z = a$. [6]

5. (*a*) Examine for consistency of the system of linear equations and solve if consistent : [6]

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$$

(*b*) Examine for linear dependence or independence the vectors (1, 1, 1, 3), (1, 2, 3, 4), (2, 3, 4, 7). Find the relation between them if dependent. [6]

(*c*) Determine the values of a, b, c when A is orthogonal where : [5]

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}.$$

Or

6. (*a*) Investigate for what values of a and b, the system of equations $2x - y + 3z = 2$, $x + y + 2z = 2$, $5x - y + az = b$ have :

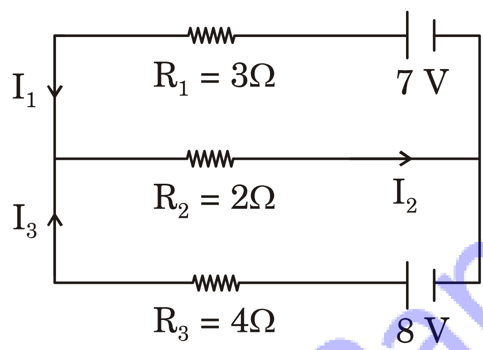
(1) No solution

(2) A unique solution

(3) An infinite number of solutions. [6]

(*b*) Examine for linear dependence or independence the vectors $x_1 = (2, 3, 4, -2)$, $x_2 = (1, 1, 2, -1)$, $x_3 = \left(\frac{-1}{2}, -1, -1, \frac{1}{2}\right)$. Find the relation between them if dependent. [6]

(*c*) Determine the currents in the network given in figure below : [5]



7. (*a*) Find the eigen values and the corresponding eigen vectors for the following matrix : [6]

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

(*b*) Verify Cayley-Hemilton theorem for $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ and use it to find A^{-1}. [6]

(*c*) Find a matrix P that diagonalizes the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$ [6]

Or

8. (*a*) Find the eigen values and the corresponding eigen vectors for the following matrix : [6]

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

(*b*) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ and use it to find A^{-1}. [6]

(*c*) Reduce the following quadratic form to the sum of the squares form : [6]

$$Q = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz.$$