Total No. of Questions—**8**] [Total No. of Printed Pages—**4**

Seat No.	

[5667]-108

F.E. EXAMINATION, 2019

ENGINEERING MATHEMATICS–I

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. **1** *or* Q. **2**, Q. **3** *or* Q. **4**, Q. **5** *or* Q. **6** and Q. **7** *or* Q. **8**.

(*ii*) Neat diagram must be drawn wherever necessary.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Find the rank of the matrix : [4]

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}.$$

(*b*) Find the eigen values and eigen vector corresponding to largest eigen value of a matrix : [4]

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}.$$

(*c*) Solve $x^5 - 1 = 0$ using DeMoivres Theorem. [4]

Or

2. (*a*) Examine for linear dependence or independence of vectors : [4]

$x_1 = (1, 1, -1), x_2 = (2, 3, -5), x_3 = (2, -1, 4).$

(*b*) If $\operatorname{cosec}(x+iy) = u+iv$, prove that : [4]

$$(u^2+v^2)^2 = \frac{u^2}{\sin^2 x} - \frac{v^2}{\cos^2 x}.$$

(*c*) Separate real and immaginary parts of $(1+i)^i$. [4]

3. (*a*) Solve any *one* : [4]

(*i*) Test for convergence the series :

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}.$$

(*ii*) Test for convergence the series :

$$\frac{1!}{1^1} + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \ldots\ldots$$

(*b*) Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of x. [4]

(*c*) Find the nth derivative of [4]

$$y = \frac{1}{(x-1)^2\,(x-2)}.$$

Or

4. (*a*) Solve any *one* : [4]

(*i*) Find the values of a and b if :

$$\lim_{x\to 0}\left(\frac{\sin x}{x^3} + \frac{a}{x^2} + b\right) = 0.$$

(*ii*) Evaluate :

$$\lim_{x\to\frac{\pi}{2}} (\sec x)^{\cot x}.$$

(*b*) Using Taylor's theorem, expand $4x^3 + 3x^2 + 2x + 1$ in ascending powers of $(x + 1)$. [4]

(*c*) If $y = \cos(m \log x)$, prove that : [4]

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + (m^2 + n^2) y_n = 0.$$

5. Solve any *two* :

(*a*) If $u = 4e^{-6x} \sin(pt - 6x)$ satisfies the partial differential equation $u_t = u_{xx}$ then find the value of ϕ. [6]

(*b*) If

$$T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2} + \frac{x^2 y}{x + y},$$

find the value of $xT_x + yT_y$. [7]

(*c*) If $u = f(x - y, y - z, z - x)$ then find the value of : [6]

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}.$$

Or

6. Solve any *two* :

(*a*) If $x = u \tan v$, $y = u \sec v$ prove that : [6]

$$(u_x)_y \cdot (v_x)_y = (u_y)_x \cdot (v_y)_x.$$

(*b*) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$ then prove that : [7]

$$u_{xx} + u_{yy} = \frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr}.$$

(*c*) If $z = f(u, v)$ where u, v are homogeneous functions of degree 10 in x, y then prove that : [6]

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 10\left(u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v}\right).$$

7. (*a*) If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$,

evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$. [4]

(*b*) Prove that the functions :

$$u = y + z, \quad v = x + 2z^2, \quad w = x - 4yz - 2y^2$$

are functionally dependent. [4]

(*c*) Find all the stationary points of the function :

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

Examine whether the function is maximum or minimum at those pionts. [5]

Or

8. (*a*) If

$$u + v = x^2 + y^2, \quad u - v = x + 2y,$$

find $\left(\dfrac{\partial u}{\partial x}\right)_y$, by using Jacobians. [4]

(*b*) The focal length of a minor is found from the formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{2}{f}$.

Find the percentage error in f given u and v are both of error 2% each. [4]

(*c*) Find the stationary points of $T(x, y, z) = 8x^2 + 4yz - 16z + 600$ if the condition $4x^2 + y^2 + 4z^2 = 16$ satisfied. [5]