Total No. of Questions—8] [Total No. of Printed Pages—4

Seat No.	

[5558]-101

F.E. EXAMINATION, 2019
ENGINEERING MATHS—I
(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*iv*) Assume suitable data, if necessary.

(*v*) Attempt Q. No. **1** *or* **2**, Q. No. **3** *or* **4**, Q. No. **5** *or* **6**, Q. No. **7** *or* **8**.

1. (*a*) Reduce the following matrix to its normal form and hence find its rank : [4]

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}.$$

(*b*) Find eigen values and eigen vector corresponding to highest eigen value of the following matrix : [4]

$$A = \begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

(*c*) Solve the equation $x^3 - 1 = 0$ by applying Demoivre's theorem. [4]

P.T.O.

Or

2. (*a*) Investigate for what values of k the system of equations $x + y + z = 1, 2x + y + 4z = k, 4x + y + 10z = k^2$ have infinite number of solutions. [4]

(*b*) Find locus of z such that : [4]

$$|z + 1| = |z - i|.$$

(*c*) If $\sin(x + iy) = u + iv$ prove that $u^2 \operatorname{cosec}^2 x - v^2 \sec^2 x = 1$ and $u^2 \sec h^2 y + v^2 \operatorname{cosec} h^2 y = 1$. [4]

3. (*a*) Solve any *one* : [4]

(*i*) Test for convergence the series $\sum_{n=1}^{\infty} \frac{2n - 1}{n(n+1)(n+2)}$.

(*ii*) Test the convergence of the series :

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots..$$

(*b*) Expand $\log(1 + x + x^2 + x^3)$ upto the term in x^8. [4]

(*c*) Find the nth derivative of $y = \frac{x}{(x-1)(x-2)(x-3)}$. [4]

Or

4. (*a*) Solve any *one* : [4]

(*i*) Evaluate :

$$\lim_{x \to 0} \frac{e^x - 1 - x}{\log(1 + x) - x}.$$

(*ii*) Evaluate :

$$\lim_{x \to 0} (\sin x)^{\tan x}.$$

(*b*) Expand $x^3 + 7x^2 + x - 6$ in powers of $x - 3$. [4]

(*c*) If : [4]

$$y = e^{\tan^{-1} x},$$

prove that :

$$(1 + x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0.$$

5. Solve any *two* :

(*a*) If $z = \tan(y + ax) + (y - ax)^{3/2}$, find the value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$. [6]

(*b*) If $x^2 = au + bv$ and $y^2 = au - bv$, then prove that : [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \cdot \left(\frac{\partial y}{\partial v}\right)_u .$$

(*c*) If $u = \sin^{-1}(x^2 + y^2)^{1/5}$, then prove that : [7]

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = \frac{2}{5} \tan u \left[\frac{2}{5} \tan^2 u - \frac{3}{5}\right].$$

Or

6. Solve any *two* :

(*a*) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$, then prove that : [7]

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + 2f = 0.$$

(*b*) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$, then prove that : [6]

$$u_{xx} + u_{yy} = f''(r) + \frac{1}{r} f'(r).$$

(*c*) If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, then prove that : [6]

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}.$$

7. (*a*) If : [4]

$$x = uv,\ y = \frac{u+v}{u-v}$$

find :

$$\frac{\partial(u, v)}{\partial(x, y)}.$$

(*b*) If $u = x + y + z, v = x^2 + y^2 + z^2, w = x^3 + y^3 + z^3$ find $\frac{\partial x}{\partial u}$. [4]

(*c*) Divide the number 120 into three parts so that the sum of their products taken two at a time shall be maximum. [5]

Or

8. (*a*) Examine for functional dependence and independence $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + xz.$ [4]

(*b*) Find the percentage error in the area of an ellipse with an error of 1% is made in measuring its major and minor axis. [4]

(*c*) Find the extreme values of $f(x, y) = x^3 + y^3 - 3axy, a > 0.$ [5]