

Total No. of Questions : 8]

SEAT No. :

P4417

[Total No. of Pages : 3

[5251]-1001

F.E.

ENGINEERING MATHEMATICS - I

(2015 Pattern)

Time : 2 Hours]

[Max. Marks : 50

Instructions to the candidates :

- 1) Solve Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7, or Q.8.
- 2) Neat diagrams must be drawn, wherever necessary.
- 3) Figures to the right indicate full marks.
- 4) Use of electronic pocket calculator is allowed.
- 5) Assume suitable data, if necessary.

Q1) a) Reduce the following matrix to its normal form and hence find the rank. [4]

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

b) Show that $A = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$ is an orthogonal matrix. [4]

c) A square lies above real axis in argand diagram, and of its adjacent vertices are the origin and the point $5 + 6i$, find the complex numbers representing other vertices. [4]

OR

Q2) a) Verify Cayley-Hamilton theorem for

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \text{ find } A^{-1} \quad [4]$$

b) If $\tan(x + iy) = i$, where x and y are real, prove that x is indeterminate and y is infinite. [4]

c) Considering the principal value, express in the form $a + ib$ the expression. $(\sqrt{i})^{\sqrt{i}}$. [4]

P.T.O.

Q3) a) Test the convergence of the series (any one) [4]

i) $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \dots + \frac{n}{1+2^n} + \dots$

ii) $\sum_{n=1}^{\infty} \frac{10n+4}{n^3}$

b) Expand $(1+x)^{1/x}$ in ascending powers of x, expansion being correct upto second power of x. [4]

c) Find n^{th} derivative of $y = \frac{2x+3}{(x-1)(x-2)}$ [4]

OR

Q4) a) Solve any one [4]

i) $\lim_{x \rightarrow \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2xe}$

ii) $\lim_{x \rightarrow 1} (1-x^2)^{\frac{1}{\log(1-x)}}$

b) Using Taylor's theorem, express $5 + 4(x-1)^2 - 3(x-1)^3 + (x-1)^4$ in ascending powers of x. [4]

c) If $y = e^{a \sin^{-1} x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ [4]

Q5) Solve any two

a) If $z^3 - zx - y = 4$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ [6]

b) If $u = \frac{xyz}{2x+y+z} + \log\left(\frac{x^2+y^2+z^2}{xy+yz}\right)$ Find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ [7]

c) If $x = u + v + w$, $y = uv + vw + uw$, $z = uvw$ and ϕ is a function of x, y, z then prove that $u \frac{\partial \phi}{\partial u} + v \frac{\partial \phi}{\partial v} + w \frac{\partial \phi}{\partial w} = x \frac{\partial \phi}{\partial x} + 2y \frac{\partial \phi}{\partial y} + 3z \frac{\partial \phi}{\partial z}$ [6]

OR

Q6) Solve any two

a) Find $\frac{dz}{dx}$ if $z = x^2y$ and $x^2 + xy + y^2 = 1$ [6]

b) If $u = \operatorname{cosec}^{-1} \sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$

Prove that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{144} [\tan^2 u + 13]$. [7]

c) If $x = \frac{r}{2} [e^\theta + e^{-\theta}]$ and $y = \frac{r}{2} [e^\theta - e^{-\theta}]$ prove that $\left(\frac{\partial x}{\partial r}\right)_\theta = \left(\frac{\partial r}{\partial x}\right)_\theta$ [6]

Q7) a) If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ [4]

b) Examine for functional dependence [4]

$$u = \sin^{-1}x - \sin^{-1}y, \quad v = x\sqrt{1-y^2} - y\sqrt{1-x^2}$$

c) Find the extreme values of the function $f(x, y) = x^2 + y^2 + 6x + 12$. [5]

OR

Q8) a) If $ux + vy = 0$, $\frac{u}{x} + \frac{v}{y} = 1$ then using Jacobian find $\left(\frac{\partial u}{\partial x}\right)_y$. [4]

b) The focal length of a mirror is found from the formula : $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$.

Find the percentage error in f if u and v are both in error by $p\%$ each. [4]

c) Find the point on the surface $z^2 = xy + 1$ nearest to the origin, by using Lagrange's method. [5]

