Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[5056]-11

F.E. (Common) EXAMINATION, 2016

ENGINEERING MATHEMATICS—I

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** or Q. No. **6** and Q. No. **7** or Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) For which values of k the system of equations : [4]

$$x + 2y - z = 0$$

$$3x + (k + 7)y - 3z = 0$$

$$2x + 4y + (k - 3)z = 0$$

will possess non-trivial solutions ?

(*b*) Find eigen values and eigen vector corresponding to the lowest eigen value for the matrix : [4]

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -1 & 3 \end{bmatrix}$$

(*c*) If

$$\frac{z-2i}{2z-1}$$

is purely imaginary, prove that the locus of z in the Argand's diagram is a circle. Find its centre and radius. [4]

Or

2. (*a*) Reduce the matrix : [4]

$$\begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 3 & 2 & 7 & 5 & 12 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

to normal form and hence find its rank.

(*b*) Prove that : [4]

$$\log(e^{i\alpha} + e^{i\beta}) = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right).$$

(c) If

$$y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

prove that : [4]

$$\tanh \frac{y}{2} = \tan \frac{x}{2}.$$

3. (a) Test the convergence of the series (any *one*) : [4]

(i) $1 + \frac{x}{2^2} + \frac{x^2}{3^2} + \frac{x^3}{4^2} + \ldots\ldots$

(ii) $\frac{1}{3} - \frac{1}{6\sqrt{2}} + \frac{1}{9\sqrt{3}} - \frac{1}{12\sqrt{4}} + \ldots\ldots$

(b) Prove that : [4]

$$\log\left[\frac{1+e^{2x}}{e^x}\right] = \log 2 + \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} + \ldots\ldots.$$

(c) Find the *n*th derivative of : [4]

$$y = \cosh(x) \,.\, \cos 3x.$$

Or

4. (a) Solve any *one* : [4]

(i) $\lim_{x \to 0} \frac{\log \sin 2x}{\log \sin x}$

(ii) $\lim_{x \to \pi/2} (\sec x)^{\cot x}.$

(*b*) Using Taylor's theorem expand $1 + 2x + 3x^2 + 4x^3$ in powers of $(x + 1)$. [4]

(*c*) If

$$y = \frac{\log x}{x},$$

using Leibnitz's theorem prove that : [4]

$$y_n = \frac{(-1)^n n!}{x^{n+1}} \left[\log x - \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \right) \right].$$

5. Solve any *two* questions :

(*a*) If

$$u = 4e^{-6x} \sin[pt - 6x]$$

satisfies the partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, then find value of p. [7]

(*b*) If

$$u = \frac{\sqrt{x^7 + y^7}}{3\sqrt{x^4 + y^4}} + \cos\left[\frac{xy + y^2}{4xy}\right] + \log\left(\frac{x}{y}\right)$$

then find value of : [6]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial y} + y \frac{\partial u}{\partial y}.$$

(*c*) If

$$u = f(r, s), \quad r = x^2 + y^2, \quad s = x^2 - y^2$$

prove that : [6]

$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 4xy\frac{\partial u}{\partial r}.$$

Or

6. Solve any *two* questions :

(*a*) If

$$u = 2x + 3y, \quad v = 3x - 2y$$

prove that : [7]

(*i*) $\left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u = \frac{13}{4}$

(*ii*) $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{4}{13}.$

(*b*) If

$$u = \tan^{-1}\left[\frac{\sqrt{x^3 + y^3}}{\sqrt{x} + \sqrt{y}}\right],$$

then prove that : [6]

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin 2u \ \sin^2 u.$$

(*c*) If $z = f(u, v)$ and if *u, v* are homogeneous functions in *x, y* of degree 10 each, then prove that : [6]

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 10\left(u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v}\right).$$

7. (*a*) If

$$x = r\cos\theta, \quad y = r\sin\theta$$

show that $JJ' = 1$. [4]

(*b*) In calculating the volume of a right circular cone, using the formula :

$$V = \frac{1}{3}\pi r^2 h,$$

errors of 2% and 1% are made in measuring the height and radius of base respectively. Find the error in the calculated volume. [4]

(*c*) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere :

$$x^2 + y^2 + z^2 = 1,$$

using Lagrange's method. [5]

Or

8. (*a*) If $u = x(1-y)$ and $v = xy$, find : [4]

$$\frac{\partial(x, y)}{\partial(u, v)}.$$

(*b*) In estimating the cost of a pile of bricks measured as $6' \times 50' \times 4'$, the tape is stretched 1% beyond the standard length. If the count is 12 bricks to 1 ft^3 and bricks cost Rs. 100 per thousand, find the approximate error in the cost. [5]

(*c*) Find the stationary values of : [4]

$$f(x, y) = x^3 y^2 (1 - x - y).$$