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S.E. (E&TC/Elect.) (II Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,
Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve any *two* of the following : [8]

(i) $(D^2 + 5D + 6) y = e^x$

(ii) $(D^2 + 4) y = \sec 2x$

(by method of variation of parameters)

(iii) $\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}$

P.T.O.

(b) Find the Fourier cosine transform of the function : [4]

$$f(x) = \begin{cases} x^2 & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$

Or

2. (a) An uncharged condenser of capacity C charged by applying an e.m.f. of value $E \sin \frac{t}{\sqrt{LC}}$ through the leads of inductance L and negligible resistance. The charge Q on the plate of condenser satisfies the differential equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

Prove that the charge at any time t is given by :

$$Q = \frac{EC}{2} \left[\sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right].$$

(b) Find the inverse z -transform (any one) : [4]

(i) $F(z) = \frac{2z^2 + 3z}{(z+2)(z-4)}, \quad |z| > 4$

(ii) $F(z) = \frac{10z}{(z-1)(z-2)}$

(by inversion integral method)

(c) Solve the following difference equation : [4]

$$f(k+2) + 3f(k+1) + 2f(k) = 0;$$

$$f(0) = 0, \quad f(1) = 2, \quad k \geq 0.$$

3. (a) Solve the differential equation $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$ to get $y(0.2)$ by using Runge-Kutta method of fourth order. ($h = 0.2$) [4]

(b) Find Lagrange's interpolation polynomial passing through set of points : [4]

x	y
0	2
1	3
2	12
5	147

(c) Find directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ towards the point $\bar{i} + \bar{j} - \bar{k}$. [4]

Or

4. (a) Prove any *one* of the following : [4]

$$(i) \quad \nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$$

$$(ii) \quad \nabla^4 e^r = e^r + \frac{4}{r} e^r.$$

(b) Show that the vector field : [4]

$$\bar{F} = (2xz^3 + 6y) \bar{i} + (6x - 2yz) \bar{j} + (3x^2z^2 - y^2) \bar{k}$$

is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(c) Compute the value of definite integral : [4]

$$\int_0^6 \frac{1}{1+x} dx$$

using Simpson's $\left(\frac{3}{8}\right)$ th rule, dividing the interval into 6 parts.

5. (a) Evaluate : [4]

$$\int_C \vec{F} \cdot d\vec{r}$$

for the field $\vec{F} = x^2 \vec{i} + xy \vec{j}$ over the region R enclosed by $y = x^2$ and the line $y = x$ using Green's theorem.

(b) Evaluate : [4]

$$\iiint_S \vec{F} \cdot \hat{n} dS$$

for $\vec{F} = 4xz \vec{i} - y^2 \vec{j} + yz \vec{k}$ and S, the surface of cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$ using Divergence theorem.

(c) Using Stokes' theorem calculate : [5]

$$\int_C 4y dx + 2z dy + 6y dz,$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$.

Or

6. (a) Find the workdone by the force field given by : [4]

$$\vec{F} = 3x^2 \vec{i} + (2xz - y) \vec{j} + z \vec{k}$$

along the curve $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$.

- (b) Evaluate : [4]

$$\iint_S \vec{F} \cdot d\vec{S}$$

for $\vec{F} = 4xz \vec{i} - y^2 \vec{j} + yz \vec{k}$ and S, the surface of the cube bounded by the planes $x = 0$, $x = 3$, $y = 0$, $y = 3$, $z = 0$, $z = 3$ by using Divergence theorem.

- (c) Evaluate : [5]

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

for $\vec{F} = y \vec{i} + z \vec{j} + x \vec{k}$, where S is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$.

7. (a) If $f(z) = u + iv$ is analytic, show that $u = c$, $v = b$ are orthogonal. [4]

- (b) Evaluate : [4]

$$\oint_C \frac{z^2 + 1}{z - 3} dz$$

where

(i) 'C' is the circle $|z - 3| = 2$

(ii) 'C' is the circle $|z| = \frac{3}{2}$.

- (c) Show that under the transformation $\omega = z + \frac{1}{z}$ family of circles $r = c$ are mapped on to family of ellipses. What happens if $c = 1$? [5]

Or

8. (a) If $f(z) = u + iv$ is analytic, show that u, v are Harmonic functions. [4]
- (b) Evaluate : [4]

$$\oint_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^4} dz$$

where 'C' is the circle $|z| = \frac{3}{2}$.

- (c) Find the bilinear transformation which maps the points $z = -1, 0, 1$ onto the points $\omega = 0, i, 3i$ respectively. [5]