Total No. of Questions—**8**] [Total No. of Printed Pages—**6**]

Seat No.	

[5152]-540

S.E. (E&TC/Electronics Engg.) (II Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS–III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 7D + 6)y = e^{2x}$

(*ii*) $(D^2 + 4)y = x \sin x$

(*iii*) $(x+2)^2 \dfrac{d^2y}{dx^2} + 3(x+2)\dfrac{dy}{dx} + y = 4 \sin \log (x+2)$.

(*b*) Find the Fourier sine transform of the function : [4]

$$f(x) = e^{-x}, \; x > 0.$$

P.T.O.

Or

2. (*a*) A circuit consists of an inductance L and condenser of capacity C in series. An alternating e.m.f. $E \sin nt$ is applied to it at time $t = 0$, the initial current and charge on the condenser being zero and $w^2 = \frac{1}{LC}$, find the current flowing in the circuit at any time for $w \neq n$. [4]

(*b*) Find inverse *z*-transform (any *one*) : [4]

(*i*) $F(z) = \dfrac{z}{(z-1)(z-3)}, \ |z| > 3$

(*ii*) $F(z) = \dfrac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$.

(by inversion integral method).

(*c*) Solve the following difference equation : [4]

$$f(k+1) + \frac{1}{4} f(k) = \left(\frac{1}{4}\right)^k ;$$

$$f(0) = 0, \quad k \geq 0$$

3. (*a*) Using fourth order Runge-Kutta method, solve the differential equation : [4]

$$\frac{dy}{dx} = x + y + xy$$

with $y(0) = 1$ to get $y(0.1)$ taking $h = 0.1$.

(*b*) Find Lagrange's interpolating polynomial passing through the set of points : [4]

x	***y***
0	3
1	4
3	12

(*c*) Find the directional derivative of : [4]

$$\phi = x^2 - y^2 + 2z^2$$

at the point (1, 2, 3) in the direction of $4\bar{i} - 2\bar{j} + \bar{k}$.

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla\left(\frac{\bar{a}.\bar{r}}{r^2}\right) = \frac{\bar{a}}{r^2} - \frac{2(\bar{a}.\bar{r})\,\bar{r}}{r^4}$

(*ii*) $\nabla\left(\bar{a}.\nabla\frac{1}{r}\right) = \frac{3(\bar{a}.\bar{r})\bar{r}}{r^5} - \frac{\bar{a}}{r^3}$

(*b*) Show that : [4]

$$\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$$

is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(*c*) By using Trapezoidal rule, evaluate : [4]

$$\int_0^2 \frac{1}{1 + x^4}\, dx$$

taking $h = \frac{1}{2}$, correct upto 3-decimal places.

5. (*a*) Evaluate $\int \overline{\mathrm{F}} \cdot d\overline{r}$ for [4]

$$\overline{\mathrm{F}} = 3x^2\,\overline{i} + (2xz - y)\overline{j} + z\overline{k}$$

along straight line joining (0, 0, 0) and (2, 1, 3).

(*b*) Evaluate : [4]

$$\iint_{\mathrm{S}} \left[(4x + 3yz^2)\overline{i} - (x^2z^2 + y)\overline{j} + (y^2 + 2z)\overline{k}\right] . d\overline{\mathrm{S}}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$.

(*c*) Apply Stokes' theorem to evaluate $\oint_{\mathrm{C}} \overline{\mathrm{F}}.d\overline{r}$ where : [5]

$$\overline{\mathrm{F}} = xy^2\,\overline{i} + y\overline{j} + xz^2\overline{k}$$

and C is the boundary of rectangle :

$x = 0$, $y = 0$, $x = 1$, $y = 2$ in $z = 0$ plane.

Or

6. (*a*) Using Green's lemma evaluate : [4]

$$\int_{\mathrm{C}} (xy - x^2)\,dx + x^2y\,dy$$

along the curve C : $x = 1$, $y = x$, $y = 0$.

(*b*) Evaluate : [5]

$$\iint (\nabla \times \overline{\mathrm{F}}) \,.\, \hat{n}\; ds,$$

where $\overline{\mathrm{F}} = (x - y)\overline{i} - (x^2 + yz)\overline{j} - 3xy^2\overline{k}$ and S is the surface of the cone $z = 4 - \sqrt{x^2 + y^2}$, above the *xoy* plane.

(c) Prove that : [4]

$$\iint_s (\phi\nabla\Psi - \Psi\nabla\phi).d\overline{S} = \iiint_v (\phi\nabla^2\Psi - \Psi\nabla^2\phi)\,dV.$$

7. (a) Show that the function : [4]

$$f(z) = u + iv$$

with constant modulus and constant amplitude is constant in each case.

(b) Evaluate : [4]

$$\oint_C \frac{4z^2 + z}{z^2 - 1}\,dz$$

where C is the circle $|z - 1| = \frac{1}{2}$.

(c) Find the bilinear transformation which maps the points :

$$z = 1,\ i,\ 2i$$

onto the points $w = -2i, 0, 1$ respectively. [5]

Or

8. (a) If [5]

$$f(z) = u + iv$$

is analytic, find $f(z)$, if

$$u - v = x^2 - y^2 - 2xy.$$

(*b*) Evaluate : [4]

$$\oint_C \frac{(z^2 + \cos^2 z)}{\left(z - \frac{\pi}{4}\right)^3} dz,$$

where C is circle $|z| = 1$.

(*c*) Find the map of the straight line $y = x$ under the transformation $w = \frac{z-1}{z+1}$. [4]