Total No. of Questions : 9]

SEAT No. :

PE4254 **[6582]-25** [Total No. of Pages : 4

S.E. (Electronics/E & TC) (Electronics & Computer/ Electronics (VLSI Design & Tech.)/E.C (A.C.T.))

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207005)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt Q.1, Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Black figures to the right indicate full marks.*
4) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

a) Lagrange's polynomial through the points

x	0	1	2
y	4	0	6

is given by

i) $y = 5x^2 - 3x + 4$

ii) $y = 5x^3 + 3x + 4$

iii) $y = 5x^2 - 9x + 4$

iv) $y = x^2 - 9x + 4$

b) The curl of the vector field

$\overline{F} = z^2\overline{i} + x^2\overline{j} + y^2\overline{k}$ at (1, 0, –1) is

i) $2\overline{j} + 2\overline{k}$

ii) $2\overline{i} + 2\overline{j} + 2\overline{k}$

iii) $\overline{j} + \overline{k}$

iv) $\overline{i} + \overline{j} + \overline{k}$

c) For $\overline{F} = y\overline{i} + z\overline{j}$ the value of $\int \overline{F}.dr$ along the curve $x = t^2$, $y = t$, $z = 2t$ from $t = 0$ to $t = 1$

i) 7/3

ii) 5/3

iii) 3/2

iv) 2/3

d) The value of $\int_c \frac{z+1}{z-3}dz$ where C is $|z-2| = 2$

i) $2\pi i$

ii) $4\pi i$

iii) $6\pi i$

iv) $8\pi i$

P.T.O.

e) In Runge-kutta method k_1, k_2, k_3, k_4 are Calculated then $y = y_0 + k$, where k is calculated from **[1]**

i) $k = \frac{1}{4}(k_1 + k_2 + k_3 + k_4)$

ii) $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

iii) $k = \frac{1}{8}(k_1 + 2k_2 + k_3 + k_4)$

iv) $k = \frac{1}{10}(k_1 + 2k_2 + 2k_3 + k_4)$

f) Vector field $\bar{F}$ is solenoidal if **[1]**

i) $\nabla \times \bar{F} = 0$

ii) $\nabla . \bar{F} = 0$

iii) $\nabla^2 \bar{F} = 0$

iv) $\bar{F} . \nabla = 0$

Q2) a) Apply Lagrange's interpolating formula to find polynomial passing through set of points. **[5]**

x	0	1	2
y	4	3	6

Also Find Value of y at $x = 1.5$.

b) Use Trapezodial rule to evaluate $\int_0^1 xe^{x^2} dx$ by taking h = 0.1 with **[5]**

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x) = xe^{x^2}$	0	0.101	0.208	0.328	0.469	0.642	0.86	1.14	1.16	2.203	2.7

c) Using Runge Kutta Fourth order method to solve $\frac{dy}{dx} = \sqrt{x+y}$, $y(0) = 1$ to find y at $x = 0.2$ taking $h = 0.2$. **[5]**

OR

Q3) a) Using Newton's Forward difference formula, For following data & find polynomial **[5]**

x	0	2	4	6	8
y	5	29	125	341	725

b) Evaluate $\int_0^{\pi/2} \frac{\sin x}{x} dx$ by using simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by dividing interval into four parts. **[5]**

c) By using Euler's method, find y at $x = 0.4$ for $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ with $h = 0.1$. **[5]**

Q4) a) Find the directional derivative of $\phi = x^2 y + yz^3$ at (1, –1, 1) along the direction $\bar{i} + 2\bar{j} + 2\bar{k}$. **[5]**

b) Show that $\bar{F} = (4xy + z^3)\bar{i} + (2x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational. Also find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. **[5]**

c) Find the angle between the tangents to the curve $\bar{r} = (t^2 + 1)\bar{i} + (4t - 3)\bar{j} + (2t^2 - 6t)\bar{k}$ at the points $t = 1$ and $t = 2$. **[5]**

OR

Q5) a) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along the tangent to the curve $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$ at $t = 0$. **[5]**

b) Find the constants a & b such that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2). **[5]**

c) $\nabla . \left(r \nabla \frac{1}{r^n} \right) = \frac{n(n-2)}{r^{n+1}}$. **[5]**

Q6) a) Using Green's theorem, show that Area bounded by a simple closed curve C is given by $\frac{1}{2}\int xdy - ydx$. Hence find the area of ellipse $x = a\cos(\theta), y = b\sin(\theta)$. **[5]**

b) Evaluate $\iint_s (x\bar{i} + y\bar{j} + z^2\bar{k}) . d\bar{s}$ where S is the curved surface of the cylinder $x^2 + y^2 = 4$, bounded by the planes $z = 0$ and $z = 2$. **[5]**

c) Apply stoke's theorem to calculate $\int_c 4y\,dx + 2z\,dy + 6y\,dz$, where C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. **[5]**

OR

Q7) a) Evaluate $\int_c \bar{F}.d\bar{r}$ for $\bar{F} = (2xy + 3z^2)\bar{i} + (x^2 + 4yz)\bar{j} + (2y^2 + 6xz)\bar{k}$ along the straight line joining (0, 0, 0) and (1, 1, 1). **[5]**

b) Show that $\iint \frac{\bar{r}}{r^3} \hat{n}\, ds = 0$. **[5]**

c) Evaluate $\iint_s \nabla \times \bar{F}.d\bar{s}$ for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$ where S is the surface of the paraboloid $Z = 1 - x^2 - y^2, Z \geq 0$. **[5]**

Q8) a) If $u = x^h - 6x^2y^2 + y^4$, then find it's harmonic conjugate 'v' such that $f(z) = u + iv$ is analytic. **[5]**

b) Evaluate $\oint_c \frac{e^z}{(z+1)^2(z-1)^2} dz$, where 'C' is the contour $|z+1| = \frac{1}{2}$, using Cauchy's integral formula. **[5]**

c) Find the bilinear transformation, which send S the points 1, i, –1 from z-plane into the points i, o, –i of the w-plane. **[5]**

OR

Q9) a) If $v = 3x^2y - y^3$, then find it's harmonic conjugate 'u'. **[5]**

b) Apply residue theorem to evaluate $\oint_c \frac{z^2 + 2z}{(z+1)^3(z^2 - 9)} dz$, where 'C' is $|z-3| = 3$. **[5]**

c) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. **[5]**

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