Total No. of Questions : 9]

SEAT No. :

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S.E. (Electronics/E & TC) (Electronics & Computer/VLSI Design & Technology/Electronics & Communication/Advanced Communication Technology)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester- III) (207005)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q1 is compulsory.*
2) *Attempt Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions.

a) The second forward difference $\Delta^2 f(x)$ for $f(x) = x^2$, $h = 2$ is given by **[2]**

i) 6 ii) 12

iii) 4 iv) 8

b) $\nabla(r^2 \log r)$ is equals to **[2]**

i) $r\bar{r}(2\log r+1)$ ii) $\bar{r}(2\log r+1)$

iii) $r(2\log r-1)$ iv) $r\log r(r+1)$

c) The residue of $f(z) = \cot z$ at $z = 0$ is **[2]**

i) 1 ii) 2

iii) π iv) 0

d) The value of $\int_C \bar{F}_0 d\bar{r}$, for $\bar{F} = x\bar{i} + y\bar{j} + z\bar{k}$ along the path $n = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$ is **[2]**

i) $\frac{2}{3}$ ii) $\frac{3}{2}$

iii) $\frac{5}{3}$ iv) $\frac{4}{3}$

P.T.O.

e) In Runge-Kutta method of fourth order, the value of K_2 is **[1]**

i) $\frac{h}{2} f(x_0, y_0)$ ii) $hf(x_0, y_0)$

iii) $hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$ iv) $f(x_0, y_0)$

f) The poles of $\frac{z^2+1}{(z+1)(z^2-9)}$ are **[1]**

i) $-1, -3, 3$ ii) $1, 3, 4$

iii) $-1, 3, 3$ iv) $1, 2, 3$

Q2) a) Find cubic polynomial by Newton's forward difference formula. Hence find $f(4)$. **[5]**

x	0	1	2	3
$f(x)$	1	2	1	10

b) Use Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\sin x}{x}\right) dx$ by dividing the interval into 4 equal parts. **[5]**

c) Using Euler's modified method, find the value of y at $x = 0.05$ given that $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ (2 iterations only). **[5]**

OR

Q3) a) The velocity distribution of fluid near a flat surface is given below **[5]**

x	0.1	0.3	0.6	0.8
v	0.72	1.81	2.73	3.47

Where x is distance from surface (mm). v is velocity (mm/sec). Use Lagrange's interpolation formula to find v at $x = 0.4$.

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ taking $h = \frac{1}{6}$ by using Simpson's $\frac{3}{8}^{th}$ rule compare with actual value. **[5]**

c) Use fourth order Runge-Kutta formula to find the value of y when $x = 1$ Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ and $y(0) = 1$ take $h = 1$. **[5]**

Q4) a) Find the directional derivative of $f(x, y, z) = x^2y + xyz + z^3$ at $(1, 2, -1)$ along normal to the surface $x^2y^3 = 4xy + y^2z$ at the point $(1, 2, 0)$. **[5]**

b) Show that the vector field $\bar{F} = (y^2 \cos x + z^2)i + (2y \sin x)j + (2xz)k$ is conservative and find scalar field ϕ. Such that $\bar{F} = \nabla\phi$. **[5]**

c) Show that $\nabla^2\left[\nabla.\left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}$. **[5]**

OR

Q5) a) Find the values of a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^2$ at $(2, 1, 1)$ has a maximum magnitude 12 in the direction parallel to x - axis. **[5]**

b) If $\bar{F}_1 = yzi + zxj + xyk$, $\bar{F}_2 = (\bar{a}.\bar{r})\bar{a}$ then show that $\bar{F}_1 \times \bar{F}_2$ is solenoidal. **[5]**

c) For scalar functions ϕ and ψ show that **[5]**

$$\nabla.(\phi\nabla\psi - \psi\nabla\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$$

Q6) a) Find the work done in moving a particle in a force field $\bar{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. **[5]**

b) Using Gauss-Divergence theorem, evaluate $\iint_s \bar{F}.\hat{n}\,ds$ where $\bar{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **[5]**

c) By using Stoke's thoerem, evaluate $\iint_s (\nabla \times \bar{F}).\hat{n}\,ds$ for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$ where s is the surface of the parabolide $z = 1 - x^2 - y^2$, $z \geq 0$. **[5]**

OR

Q7) a) A vector field is given by $\bar{F} = \cos y\,\bar{i} + x(1 - \sin y)\,\bar{j}$. By using Green's theorem, evaluate $\int_c \bar{F}.d\bar{r}$ where c is the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$, $z = 0$. **[5]**

b) Show that $\iiint_v \dfrac{dv}{r^2} = \iint_s \dfrac{\bar{r}.\hat{n}}{r^2}\,ds$. **[5]**

c) By using Stoke's theorem, evaluate $\int_c \bar{F}.d\bar{r}$ where $\bar{F} = \sin z\,\bar{i} + \cos x\,\bar{j} + \sin y\,\bar{k}$ and 'c' is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$ & $z = 3$. **[5]**

Q8) a) If $v = 3x^2y - y^3$, then find it's harmonic conjugate u. **[5]**

b) Evaluate $\oint_c \dfrac{z^2+1}{z-2}\,dz$, where c is the circle $|z - 2| = 1$, by using Cauchy's Integral formula. **[5]**

c) Find the bilinear transformation which maps the points $z = 1, i, 2i$ on the points $w = -2i, 0, 1$ respectively. **[5]**

OR

Q9) a) If $u = x^4 - 6x^2y^2 + y^4$ then find v such that $f(z) = u + iv$ is analytic & determine $f(z)$ in terms of z. **[5]**

b) Evaluate $\oint_c \dfrac{\sin(\pi z^2) + 2z}{(z-2)}\,dz$, where c is the circle $|z| = 4$, by Residue theorem. **[5]**

c) Find the map of the straight line $y = x$ under the transformation $w = \dfrac{z-1}{z+1}$. **[5]**