Total No. of Questions : 9]

SEAT No. :

P1484

[Total No. of Pages : 4

[6002]-111

S.E. (Electronics/E&Tc/Electronics &Computer)

ENGINEERING MATHEMATICS-III

(2019 Pattern) (Semester-III) (207005)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*
7) *Write numerical calculations correct upto four decimal places.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

a) For $\bar{F} = x^2 i + xyj$, the value of $\int_c \bar{F} \cdot \overline{dr}$ for the curve $y=x$ joining the points (0,0) and (1,1) is .

i) 1

ii) $\frac{1}{3}$

iii) $\frac{3}{2}$

iv) $\frac{2}{3}$

b) The curl of vector field $\bar{F} = x^2 yi + xyzj + z^2 yk$ at the point (0,1,2) is

i) $4i - 2j + 2k$

ii) $4i + 2j + 2k$

iii) $4i + 2k$

iv) $2i + 4k$

c) The poles of $\frac{1}{z^2 + 1}$ are

i) $i, -i$

ii) $1, -1$

iii) $1, i$

iv) $1, -i$

d) Given $\frac{dy}{dx} = x + y^2$, $x = 0$, $y = 1$, $h = 0.2$ k_1 as defined in Runge-Kutta method is given by

i) 0.1

ii) 0.4

iii) 0.3

iv) 0.2

P.T.O.

e) if ∇ is the backward difference operator the $\nabla\ f(x)$ is equal to

i) $f(x) - f(x-h)$ ii) $f(x+h) - f(x)$

iii) $f(x+h)$ iv) $f(x-h)$

f) If $f(z)$ is analytic on and within the closed contour C then $\oint_c f(z)dz =$

[Given $r_1, \ldots\ldots, r_n$ are residues at poles]

i) $2\pi i$ ii) $r_1 + r_2 + \text{-----} + r_n$

iii) 0 iv) $2\pi i(r_1 + r_2 + \ldots\ldots r_n)$

Q2) a) Find Langrange's interpolation polynomial passing through the following set of points. **[5]**

x	0	1	2
y	4	3	σ

b) By Trapezoidal Rule, find the value of $\int_0^1 \frac{1}{1+x^2}dx$ by taking h=0.25. **[5]**

c) Use Runge-kutta method of fourth order to obtain the numerical solution of $\frac{dy}{dx} = x^2 + y^2,\ y(1) = 1.5$ in the interval (1,1.1) with h=0.1. **[5]**

OR

Q3) a) Find value of y for x=0.5 using Newton's forward difference formula for following data **[5]**

x	0	1	2	3	4
y	1	5	25	100	250

b) Use simpson's $\left(\frac{1}{3}\right)^{rd}$ rule with four intervals to find value of $\int_1^2 \frac{1}{x}dx$. **[5]**

c) Use modified Euler's method to find the value of y satisfying the equation $\frac{dy}{dx} = \log_e(x+y), y(1) = 2$ for x=1.2 correct upto four decimal places by taking h=0.2. **[5]**

Q4) a) Find the directional derivative of the function $\phi = xy^2 + yz^3$ at (1,–1,1) towards the point (2,1,–1). **[5]**

b) Show that the vector field **[5]**

$\overline{F} = (y\sin z - \sin x)\overline{i} + (x\sin z + 2yz)\overline{j} + (xy\cos z + y^2)\overline{k}$ is irrotational & find scalar function & such that $\overline{F} = \nabla\phi$

c) If $\overline{r}.\frac{d\overline{r}}{dt} = 0$ then show that $\overline{r}$ has constant magnitude. **[5]**

OR

Q5) a) Find the directional derivative of the function $\phi = e^{2x-y-z}$ at (1,1,1) in the direction of vector $-\overline{i} + 2\overline{j} + \overline{k}$. **[5]**

b) If $\rho\overline{E} = \nabla\phi$ then prove that $\overline{E}.\text{curl}\,\overline{E} = 0$ **[5]**

c) Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ **[5]**

Q6) a) Use Green's theorem to evaluate $\int_c \overline{F}.d\overline{r}$ **[5]**

where

$\overline{F} = (2x - \cos y)i + x(4 + \sin y)j$, C is the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1, z = 0$

b) Verify stoke's theorem for $\overline{F} = xy^2 i + yj + xz^2 k$ for the surface of rectangular lamina bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$, $z = 0$. **[5]**

c) Evaluate $\iint_s \overline{r}.\hat{n}ds$ over the surface of a sphere of radius 1 with centre at origin. **[5]**

OR

Q7) a) Using Green's theorem, show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int[xdy - ydx]$. Hence find area of the ellipse $x = 2\cos\theta, y = 3\sin\theta$. **[5]**

b) Using divergence theorem, show that $\iiint\limits_v \frac{1}{r^2} dv = \iint\limits_s \frac{1}{r^2} \bar{r} . d\bar{s}$ **[5]**

c) Verify stokes theorem for $\bar{F} = -y^3 i$ and the closed curve c is the boundary of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **[5]**

Q8) a) If $f(z) = u + iv$ is analytic, find $f(z)$ if $u-v = (x-y)(x^2 + 4xy + y^2)$ **[5]**

b) Evaluate $\oint\limits_c \frac{4z^2 + z}{(z-1)^2} dz$ where c is the contour $|z-1|=2$ by using cauchy integral formula. **[5]**

c) Find the bilinear transformation which maps the points 1, i, -1 from z–plane into the points i, 0, –i of w-plane. **[5]**

OR

Q9) a) If $u = 3x^2 - 3y^2 + 2y$ find v such that $f(z) = u+iv$ is analytic. Determine $f(z)$ in terms of z. **[5]**

b) Evaluate $\oint\limits_c \frac{z+2}{z^2+1} dz$ where c is $|z-1| = \frac{1}{2}$ by Residue theorem. **[5]**

c) Show that the image of line parallel to x-axis are mapped onto hyperbola in w-plane under the transformation $w = \sin hz$. **[5]**

● ● ●