Total No. of Questions : 9]

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S.E. (Electronics/E&TC) (Electronics & Computer) ENGINEERING MATHEMATICS - III (2019 Pattern) (Semester - III) (207005)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory.*
2) *Attempt Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*
7) *Write numerical calculations correct upto four decimal places.*

Q1) Write the correct option for the following multiple choice questions :

i) The divergence of vector field

$\vec{F} = 3x^2\bar{i} + 3y^2\bar{j} + 2xz\bar{k}$ at point (1,1,1) is **[2]**

a) 14 b) 2

c) 12 d) 8

ii) If $f(x) = x^2$, $h = 1$, $\Delta\nabla f(x)$ is given by **[2]**

a) –2 b) 1

c) 2 d) –1

iii) The value of $\int_C \frac{z^2+1}{z-2} dz$ where C is $|z| = 1$, **[2]**

a) 0 b) $2\pi i$

c) $4\pi i$ d) $\frac{\pi i}{2}$

P.T.O.

iv) By Gauss - Divergence theorem $\iint_S \bar{r}_0\, \hat{n} ds$ is equal to **[2]**

a) $3\iiint_v dv$ b) $\iiint_v \frac{1}{r^2} dv$

c) $\iiint_v dv$ d) 0

v) Inverse shifting operator is equivalent to. **[1]**

a) $1 - \delta$ b) $1 + \delta^2$

c) $1 + \delta$ d) $1 - \nabla$

vi) If $f(z)$ is analytic on and within a closed conter C then by Cauchy's Integral theorem $\oint_C f(z)dz$ is equal to **[1]**

a) $2\pi i$ b) πi

c) 0 d) 1

Q2) a) Using Newton's forward difference formula, find a polynomial passing through the points (0,1), (1,1), (2,7), (3,25), (4,61), (5,121). Hence find y and $\frac{dy}{dx}$ at $x = 0.5$. **[5]**

b) The speed (km/hr) of a train which starts from rest is given by the following table, the time being recorded in minutes. **[5]**

t (minutes)	0	2	4	6	8	10	12	14	16	18	20
v = ds/dt (km/hr)	0	10	18	25	29	32	20	11	5	2	0

Find approximately the total distance run in 20 minutes using Simpson's $\frac{1}{3}$rd rule.

c) Determine using modified Euler's method the value of y at $x = 0.1$, given **[5]**

$$\frac{dy}{dx} = x^2 + y, y(0) = 1.$$

Take h = 0.1. (Two iterations only)

OR

Q3) a) Given the table of square roots, calculate the value of $\sqrt{155}$ by Newton's backward difference formula. **[5]**

x	150	152	154	156
$y=\sqrt{x}$	12.247	12.329	12.410	12.490

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule taking h = $\frac{1}{4}$. **[5]**

c) Using fourth order Range-Kutta method, solve $\frac{dy}{dx}=\sqrt{(x+y)},\ \ y(0)=1,$ to find y at x = 0.2 taking h = 0.2. **[5]**

Q4) a) Find angle between the normal to the surface $xy = z^2$ at (1,4,2) and (–3,–3,3). **[5]**

b) Find the directional derivative of $\varphi = x^2 - y^2 - 2z^2$ at the point p(2,–1,3), in the direction PQ where Q is (5,6,4). **[5]**

c) Show that the vector field $\bar{F} = (8xy + z^4)\ \bar{i}\ (4x^2 - 2)\ \bar{j}\ + (4xz^3 - y)\ \bar{k}$ is irrotational. Find scalar potential function φ. **[5]**

OR

Q5) a) Find directional derivative of $\varphi = e^{2x}\cos(yz)$ at the origin in the direction tangent to the curve $\bar{r}\ = a\sin t +\ \bar{i}\ + a\cos t\ \bar{j}\ + at\ \bar{k}$ at $t = \frac{\pi}{4}$. **[5]**

b) Prove that $\bar{b}\times\nabla(\bar{a}\cdot\nabla\log r)=\frac{\bar{b}\times\bar{a}}{r^2}-\frac{2(\bar{a}\cdot\bar{r})}{r^4}(\bar{b}\times\bar{r}).$ **[5]**

c) Find angle between the tangents to the curve $\bar{r}=t^2\bar{i}+2t\bar{j}-t^3\bar{k}$ at the points t = 1 and t = –1. **[5]**

Q6) a) Apply Green's theorem to evaluate :

$$\int_C (x^2 dx + xydy)$$

Where C is the curve of region enclosed by $y = x^2$ and the line $y = x$. **[5]**

b) Using Gauss - Divergence theorem, evaluate : **[5]**

$$\iint_S (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}) \circ d\bar{s}$$

Over the surface of $x^2 + y^2 + z^2 = 1$.

c) Using Stoke's theorem, evaluate : **[5]**

$$\iint_s (\nabla \times \bar{F}) \circ \hat{n}\, ds$$

for $\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\,\bar{j} + (2xz + z^2)\bar{k}$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above the XOY - plane.

OR

Q7) a) Find the work done in moving a particle once around the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ z = 0$ under the field of force given by

$\bar{F} = (2x - y + z)\bar{i} + (x + y - z^2)\bar{j} + (3x - 2y + 4z)\bar{k}$. **[5]**

b) Using Gauss - Divergence Theorem, show that **[5]**

$$\iiint_V \frac{1}{r^2} dV = \iint_S \frac{\bar{r}}{r^2} \circ \hat{n}\ ds$$

c) Using stoke's theorem, evaluate **[5]**

$$\iint_s (\nabla \times \bar{F}) \circ d\bar{s}$$

Where $\bar{F} = (x^3 - y^3)\bar{i} - xyz\bar{j} + y^3\bar{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$.

Q8) a) If $u = \frac{1}{2} \log (x^2 + y^2)$, find V such that $f(z) = u + iv$ is analytic function. Express $f(z)$ in terms of z. **[5]**

b) Use Cachy's integral formula to evaluate $\oint_C \frac{e^z}{z+2} dz$ where C is the circle $|z + 2| = 2$. **[5]**

c) Find the bilinear transformation which maps the points 0,1,2 from z plane on to the points 1, $\frac{1}{2}$, $\frac{1}{3}$ of the W - plane. **[5]**

OR

Q9) a) Show that the analytic function $f(z)$ with constant modulus is constant. **[5]**

b) Use residue theorem to evaluate $\oint_C \frac{e^z}{(z+1)(z+2)} dz$ where C is the contour $|z + 1| = \frac{1}{2}$. **[5]**

c) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find centre, semi-major and semi-minor axes of ellipse **[5]**

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