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S.E. (Electronics /E&TC/Electronics & Comp. Eng./Electronics Engg.(VLSI Design & Tech.)/Electronics & Comm.(A.C.T.))

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207005)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*
7) *Write numerical calculations correct upto four decimal places.*

Q1) Write the correct option for the following multiple choice questions.

i) If $\phi = x^2 - y^2 + 2z^2$ then $\nabla\phi$ at the point (1, 2, 3) is ______ **[2]**

a) $2i - 4j - 12k$ b) $2i - 4j + 12k$

c) $2i + 4j + 12k$ d) $2i + 4j - 12k$

ii) If $f(x) = x^2$, $h = 2$ then $\Delta^2 f(x)$ is given by **[2]**

a) 6 b) 12

c) 4 d) 8

iii) If $f(z) = \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)}$ then residue of f(z) at z = 2 is _____ **[2]**

a) 0 b) – 1

c) 1 d) π

P.T.O.

iv) The work done in moving a particle in a force field $\bar{F} = (2x+y)\bar{i} + (3y-x)\bar{j}$ along the curve $x = 3t$, $y = 2t$ from $t = 0$ to $t = 1$, is __________ **[2]**

a) 15
b) 12
c) 14
d) 0

v) Shifting operator E is equivalent to _________. **[1]**

a) $1 - \delta$
b) $1 + \Delta$
c) $1 + \delta^2$
d) $2 - \delta$

vi) If f(z) is analytic on and within a closed contour C and if 'a' is any point within C then by Cauchy's integral formula $\oint \frac{f(z)}{z-a} dz$ is **[1]**

a) 0
b) $2\pi i f(a)$
c) $\frac{2\pi i}{n!} f^n(a)$
d) πi

Q2) a) State Lagrange's interpolation formula. Use this formula to find y when $x = 5$ from the following data. **[5]**

x	2	3	4	7
y	4	8	16	128

b) A curve is drawn to pass through the points given by the following table. **[5]**

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the X-axis and the ordinates $x = 1$, $x = 4$ by Simpson's $\frac{1}{3}^{rd}$ rule.

c) Use Euler's method to solve **[5]**

$$\frac{dy}{dx} = 1 + xy,\ y(0) = 1$$

and tabulate y for $x = 0$ to $x = 0.3$. Take $h = 0.1$

OR

Q3) a) Given sin 45° = 0.7071, sin 50° = 0.7660, sin 55° = 0.8192, sin 60° = 0.8660, Find sin 58° using Newton's backward difference formula. **[5]**

b) Given $\frac{dy}{dx} = x^2 - y, y(0) = 1$, find y(0.1) using Runge-Kutta method of fourth order (Take h = 0.1) **[5]**

c) Use Trapezoidal rule to evaluate $I = \int_{-3}^{3} x^4 dx$ using six equal subintervals. **[5]**

Q4) a) Find the directional derivative of $\phi = xy + yz + zx$ at (1, 1, 1) along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$ **[5]**

b) Show that the vector field

$\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational. Find scalar potential function ϕ such that $\bar{F} = \nabla\phi$ **[5]**

c) Find angle between tangents to the curve $x = t, y = t^2, z = t^3$ at $t = 1$ and $t = -1$. **[5]**

OR

Q5) a) Find the directional derivative of $\phi = xyz$ at (1, 2, –1) in the direction normal to the surface $x\log z - y^2 = -4$ at (–1, 2, 1) **[5]**

b) Prove that $\nabla^2\left[\nabla \cdot \frac{\bar{r}}{r^2}\right] = \frac{2}{r^4}$ **[5]**

c) The position vector of a particle at time t is

$\bar{r} = \cos(t-1)\bar{i} \sin h(t-1)\bar{j} + mt^3\bar{k}$. Find the condition on m. So that at any time t = 1 the acceleration is normal to the position vector. **[5]**

Q6) a) Apply Green's Theorem to evaluate $\int_C (3y\,dx + 2x\,dy)$

where C is boundary of $0 \le x \le \pi$, $0 \le y \le \sin x$ **[5]**

b) Using Gauss-divergence theorem, prove that

$$\iint_S (\phi\nabla\psi - \psi\nabla\phi)\cdot d\bar{s} = \iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV$$ **[5]**

c) Using Stoke's theorem, evaluate : **[5]**

$$\int_C [3(x-y)dx + 2xzdy + xydz]$$

Where C is the curve of intersection of paraboloid $x^2 + y^2 = 2z$ and the plane $z = 2$

OR

Q7) a) Evaluate $\int_C \bar{F}\cdot d\bar{r}$, $\bar{F} = (2x + y^2)\bar{i} + (3y - 4x)\bar{j}$ where C is the parabolic arc $y = x^2$ joining (0, 0) & (1, 1) **[5]**

b) Using Gauss-Divergence Theorem, evaluate $\iint_S \bar{F}\cdot\hat{n}\,ds$ for $\bar{F} = 4xz\,\bar{i} - y^2\,\bar{j} + yz\,\bar{k}$ where S is the surface of the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$ **[5]**

c) Evaluate $\iint_S (\nabla \times \bar{F})\cdot d\bar{s}$ for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$ where S is the surface of the paraboloid $z = 1 - x^2 - y^2, z \geq 0$ **[5]**

Q8) a) If $u = x^4 - 6x^2y^2 + y^4$, find V such that $f(z) = u + iv$ is analytic function. Express $f(z)$ in terms of z. **[5]**

b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^z}{z+2}dz$ where C is the circle $|z + 2| = 2$ **[5]**

c) Find the bilinear transformation which maps the points 0, –1, i of the z-plane onto the points 2, ∞, $\frac{1}{2}(5 + i)$ of the W-plane. **[5]**

OR

Q9) a) Show that analytic function f(z) with constant amplitude is constant. **[5]**

b) Apply residue theorem to evaluate $\oint_C \frac{4z^2+z}{z^2-1}dz$ where C is the contour $|z-| = \frac{1}{2}$ **[5]**

c) Show that the transformation $W = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find centre, semi-major and semi-minor axes of ellipse. **[5]**