Total No. of Questions : 9]

SEAT No. :

PE4249

[Total No. of Pages : 4

[6582]-20

S.E. (Electrical)

ENGINEERING MATHEMATICS-III

(2019 Pattern) (Semester-III) (207006)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No.1 is compulsory.*
2) *Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q9.*
3) *Figures to the right indicates full marks.*
4) *Neat diagrams must be drawn wherever necessary.*
5) *Use of electronic pocket calcualtor is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option:

a) The Z-transform of $f(k) = 1$; $k \geq 0$ is _____. **[2]**

i) $\frac{z}{1-z}; |z|<1$ ii) $\frac{1}{z-1}; |z|>1$

iii) $\frac{z}{z-1}; |z|>1$ iv) $\frac{1}{1-z}; |z|<1$

b) The standard deviation and arithmetic mean of a distribution are 12 and 45.5 respectively. Then, the coefficient of variation is ______. **[2]**

i) 26.37 ii) 32.43

iii) 379.16 iv) 22.15

c) The value of $\nabla \log r =$ _______. **[2]**

i) $\frac{\bar{r}}{r^2}$ ii) $\frac{1}{r^2}$

iii) $\bar{r}$ iv) $\frac{\bar{r}}{r}$

d) If 'C' is the circle $|z-1| = 1$, then $\oint_C \frac{z}{z-1} dz =$ _____. **[2]**

i) 0 ii) π

iii) πi iv) $2\pi i$

P.T.O.

e) For a Poisson distribution, n = 100 and P = 0.01 then P (x = 0) = _____. **[1]**

i) $\frac{4}{e}$ ii) $\frac{3}{e}$

iii) $\frac{2}{e}$ iv) $\frac{1}{e}$

f) The Fourier cosine transform of the function $f(x)$ defined in (0, ∞) is $F_c(\lambda)$ = ______. **[1]**

i) $\frac{1}{2\pi}\int_0^\infty f(x)\cos\lambda dx$ ii) $2\int_{-\infty}^{\infty} f(x)e^{i\lambda x}dx$

iii) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(u)e^{i\lambda u}du$ iv) $\int_0^\infty f(u)\cos\lambda u\, du$

Q2) a) Solve any one. **[5]**

i) Find z-transform of $f(k)=\cos\frac{\pi}{2}k;\ k\geq 0$

ii) Find inverse z-transform of $f(z)=\frac{1}{(z+2)(z+5)}, |z|>2$

b) Solve $f(k+1)+\frac{1}{2}f(k)=\left(\frac{1}{2}\right)^k, f(0)=0, k\geq 0$ **[5]**

c) Find the Fourier cosine transform of **[5]**

$$f(x)=\begin{cases}0 & , \quad x<0\\ \pi e^{-x}, & \quad x>0\end{cases}$$

OR

Q3) a) Solve any one. **[5]**

i) Find z-transform of $f(k)=k2^k,\ k>0$

ii) Find inverse z-transform of $f(z)=\left(\frac{1}{z-5}-\frac{1}{z-4}\right);\ 4<|z|<5$

b) Find the Fourier cosine transform of $F(x)=e^{-2x}-e^{-3x},\ \ x>0$. **[5]**

c) Solve the integral equation. **[5]**

$$\int_0^\infty F(x)\cos\lambda x\, dx=\begin{cases}1-\lambda\ , & 0\leq\lambda\leq 1\\ 0 & , \quad \lambda\geq 1\end{cases}$$

Q4) a) The four moments of a distribution about the value 10 are 3, 40, 100 and 400 respectively. Obtain the first four central moments, β_1 and β_2. **[5]**

b) Find the correlation coefficient between the load (X) and extension(Y) on five occasions of a spring as given in the following data. **[5]**

X	1	5	7	3	9
Y	5	30	50	15	75

c) A fair coin is tossed five times. What is the probability of getting at least two heads? **[5]**

OR

Q5) a) The two regression lines of a bivariate data are $3x + 2y = 26$ and $6x + y = 31$. Find : **[5]**

i) The mean values of x and y

ii) The correlation coefficient between x and y.

b) On an average, 120 cars per hour pass a specified point on a particular road. Using poisson distribution, find the probability that at most one car passes the point in a minute. **[5]**

c) The lifetime of a certain component has a normal distribution with mean 400 hours and standard deviation 50 hours. Assuming normal distribution, determine approximately the expected number of components in a sample of 1,000 components whose lifetime lies between 340 hours to 465 hours. [Given: Z_1=1.2, A_1=0.3849; Z_2=1.3, A_2=0.4032] **[5]**

Q6) a) Find the directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) towards the point (α, 1, –1). **[5]**

b) Show that: **[5]**

$\overline{F} = (y\sin z - \sin x)\overline{i} + (x\sin z + 2yz)\overline{j} + (xy\cos z + y^2)\overline{k}$ is irrotational.

Find the scalar ϕ such that $\overline{F} = \nabla\phi$.

c) If $\overline{F} = (2x + y^2)\overline{i} + (3y - 4x)\overline{j}$ then evaluate $\int_C \overline{F}\cdot d\overline{r}$ around the parabolic arc $y^2 = x$ joining (0, 0) and (1,1). **[5]**

OR

Q7) a) Find the directional derivative of $\phi = 3\log(x+y+z)$ at (1, 1, 1) in the direction of tangent to the curve. $x = b\sin t$, $y = b\cos t$, $z = bt$, at $t = 0$. **[5]**

b) Prove (any one). **[5]**

i) $\nabla\left(\bar{a}\cdot\nabla\frac{1}{r}\right) = \frac{3(\bar{a}\cdot\bar{r})\bar{r}}{r^5} - \frac{\bar{a}}{r^3}$

ii) $\nabla^2\left(\nabla\cdot\frac{\bar{r}}{r^2}\right) = \frac{2}{r^4}$

c) Using Green's theorem $\oint_C \left(xy - y^2\right)dx + x^2y\,dy$ along the closed curve formed by $y = 0$, $x = 1$ and $y = x$. **[5]**

Q8) a) If $f(z) = u + iv$ is analytic function and $V = \frac{-y}{x^2+y^2}$ then find its harmonic conjugate u and express $f(z)$ in terms of z. **[5]**

b) Evaluate $\oint_C \frac{z^2}{(z-1)^2}dz$ where 'C' is $|z|=2$ using Cauchy's integral formula. **[5]**

c) Find the bilinear transformation which sends the points 1, i, –1 from z-plane into the points i, 0, $-i$ of the w-plane. **[5]**

OR

Q9) a) If $f(z) = u + iv$ is analytic function find $f(z)$ if $u + v = e^{-x}(\cos y - \sin y)$ **[5]**

b) Evaluate $\oint_C \frac{z^2}{(z-1)(z-2)^2}dz$, where 'C' is the circle $|z|=3$, using Cauchy's residue theorem. **[5]**

c) Show that the transformation $w = z + \frac{1}{z}$ maps the circle $|z|=2$ into an ellipse. **[5]**

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