Total No. of Questions : 9]

SEAT No. :

PD-4056

[Total No. of Pages : 5

[6402]-15

S.E. (Electrical Engineering)

ENGINEERING MATHEMATICS -III

(2019 Pattern) (Semester-III) (207006)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Question no. 1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7 and Q.8 or Q.9.*
3) *Assume Suitable data if necessary.*
4) *Neat diagram must be drawn whenever necessary.*
5) *Figures to the right side indicate full marks.*
6) *Use of electronic pocket Calculator is allowed.*

Q1) a) A throw is made with two dices. The probability of getting a score of at least 10 points is, **[1]**

i) 1/6

ii) 1/4

iii) 1/12

iv) 5/6

b) Analytic function with constant amplitude is, **[1]**

i) function of x

ii) function of y

iii) function of x,y

iv) constant

P.T.O.

c) $\nabla^2\left(\frac{1}{r^2}\right)$ is equal to **[2]**

i) $\frac{1}{r^3}$ ii) $\frac{2}{r^4}$

iii) $\frac{-2}{r^4}\bar{r}$ iv) $\frac{6}{r^4}$

d) The mean and variance of binomial distribution are 5/4 and 15/16 respectively. Probability p of success in a single trial is equal to, **[2]**

i) 1/4 ii) 15/16

iii) 3/4 iv) 1/2

e) Let $I=\oint_c \frac{z}{(z-1)(z-2)}dz$ the residue of $f(z)$ at the pole $z=1$ is, **[2]**

i) 2 ii) 1/2

iii) 3 iv) 1/3

f) If $f(k) = k,\ k \geq 0$ then $z\ \{f(k)\}$ is given by **[2]**

i) $\frac{z}{(z-1)^2}, |z|>1$ ii) $\frac{(z-1)^2}{z^2}, |z|>1$

iii) $\frac{(z+1)^2}{z^2}, |z|>1$ iv) $\frac{z^2}{(z+1)^2}, |z|>1$

Q2) a) Find the Fourier transform of $f(x)=e^{-|x|}$. **[5]**

b) Attempt any one: **[5]**

i) Find Z-transform of $f(k)=\frac{\sin ak}{k}, k>0$.

ii) Find inverse Z-transform of $f(Z)=\frac{Z^2}{\left(Z-\frac{1}{2}\right)\left(Z-\frac{1}{3}\right)}, |Z|>\frac{1}{2}$

c) Obtain $f(k)$ given that

$f(k+1)+\frac{1}{2}f(k)=\left(\frac{1}{2}\right)^k, k\geq 0, f(0)=0$. **[5]**

OR

Q3) a) Attempt any one: **[5]**

i) Find Z- transform of $f(k) = 2^k\ (0)\ (3k+2),\ k \geq 0$

ii) Find inverse Z-transform of $f(Z) = \dfrac{Z}{(Z-2)(Z-3)} : 2 < |Z| < 3$

b) Find Fourier integral representation of the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \dfrac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$ **[5]**

c) Using inverse sine transform. Find $f(x)$ if $F_s(\lambda) = \dfrac{1}{\lambda} e^{-a\lambda}$ **[5]**

Q4) a) The first four moments about the value 2 are 2,20,40 and 50. Find first four moments about mean, coefficient of skewness and Kurtosis. **[5]**

b) Find correlation coefficient for following distribution, **[5]**

x	5	7	9	11	13
y	9	6	12	3	15

c) The probability of a man hitting a target is 2/3. If he fires 5 times, what is the probability of his hitting the target at most thrice? **[5]**

OR

Q5) a) Given the information; and if the correlation coefficient is 0.9, find the line of regression of x on y & estimate y for $x = 10$. **[5]**

	x	y
A. mean	8.2	12.4
S.D.	6.2	20

b) Between 2 & 3 p.m. the average number of phone calls per minute coming into the company are 2. Find the probability that during one particular minute there will be

i) No call at all

ii) 3 or less calls. **[5]**

c) If X is normally distributed and the mean of X is 15, standard deviation is 5. Determine the probability of

i) $0 \le X \le 10$

ii) $X \ge 25$

[Given : A (1) = 0.3413, A (2) = 0.4772, A(3) = 0.4987] **[5]**

Q6) a) Find directional derivative of $\phi = x^2 - y^2 - 2z^2$ at the point (2,–1,3) towards the point [5,6,4] **[5]**

b) Show that the vector field

$\bar{F} = ye^{xy}\cos z\,\hat{i} + xe^{xy}\cos z\,\hat{j} - e^{xy}\sin z\,\hat{k}$ is irrotational and find scalar field such that $\bar{F} = \nabla\phi$. **[5]**

c) Evaluate $\int_C \bar{F} \bullet d\bar{r}$ where $\bar{F} = (2xy + 3z^2)\hat{i} + (x^2 + 4yz)\hat{j} + (2y^2 + 6xz)\hat{k}$ and C is the curve $x = y = z$ from (0,0,0) to (1,1,1). **[5]**

OR

Q7) a) Find Directional derivative of $\phi = e^{2x}\cos yz$ at the origin in the direction tangent to the curve $x = a\,sint,\ y = acost,\ z = at,$ at $t = \pi/4$ **[5]**

b) Show that (any one)

i) $\nabla^4\left(r^2 \log r\right) = 6/r^2$

ii) $\nabla\left(\frac{\bar{a} \bullet \bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \bullet \bar{r})}{r^{n+2}}\bar{r}$ **[5]**

c) Using Green's theorem evaluate $\oint_C (xy - x^2)\,dx + x^2 y\,dy$ along the closed curve bounded by $y = 0 \mid x = 1$ and $y = x$ **[5]**

Q8) a) Let $f(Z) = u(r,\theta) + iv(r,\theta)$ be an analytic function and $u = -r^3 \sin 3\theta$, then construct the corresponding analytic function F(Z) in terms or Z. **[5]**

b) Evaluate $\int_C \frac{Z}{(Z^2 - 3Z + 2)}\,dZ$ where C is $|Z - z| = \frac{1}{2}$. **[5]**

c) Find the bilinear transformation which maps the points Z = 0,–1,i onto $\omega = i, 0, \infty$. **[5]**

OR

Q9) a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ & $f(Z) = u + iv$ is an analytic function of $Z = x + iy$, find $f(Z)$ in terms of Z. **[5]**

b) Evaluate $\int_C \frac{4-3z}{Z(Z-1)(Z-2)} dZ$ where C is the circle $|Z| = \frac{3}{2}$. **[5]**

c) Find the bilinear transformation which maps the points 1,i,–1 from Z-plane onto the points i,0,–i of ω plane. **[5]**