

Total No. of Questions : 9]

SEAT No. :

PC2796

[Total No. of Pages : 4

[6352]-20

S.E. (Electrical)

ENGINEERING MATHEMATICS - III
(2019 Pattern) (Semester - II) (207006)

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Question No. 1 is compulsory.
- 2) Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.
- 3) Use of electronic pocket calculator is allowed.
- 4) Figures to the right indicate full marks.
- 5) Assume suitable data if necessary.
- 6) Neat diagrams must be drawn wherever necessary.

Q1) Write the correct option :

a) Z transform of $f(k) = \left(\frac{1}{2}\right)^k, k \geq 0$ is [2]

i) $\frac{z}{z-2}$

ii) $\frac{2z}{2z-1}$

iii) $\frac{2z}{z-2}$

iv) $\frac{1}{z-2}$

b) The second and fourth moment of distribution about mean are 16 and 43.2 respectively then the coefficient of kurtosis β_2 is equal to [2]

i) 0.1687

ii) 2.262

iii) 0.3703

iv) 0.5617

c) The value of $\Delta(r^2)$ is equal to [2]

i) \bar{r}

ii) 0

iii) $2\bar{r}$

iv) 1

P.T.O.

Q4) a) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Obtain the first four central moments, β_1 and β_2 . [5]

b) Find the correlation coefficient between the number of hours studied (X) and marks obtained out of 100(Y) in Mathematics of five students as given in the following data. [5]

X	50	65	60	35	40
Y	70	85	65	50	55

c) 20% bolts produced by a machine are defective. Determine the probability that out of 5 bolts chosen at random, at the most one is defective. [5]

OR

Q5) a) If the two lines of regression are $9x + y - \lambda = 0$ and $4x + y = \mu$ and the means of x and y are 2 and -3 respectively, find the values of λ , μ and the correlation coefficient between x and y . [5]

b) On an average, there are two typing mistakes on a page of a book. Using poisson distribution, find the probability that a randomly selected page from the book has at the most one typing mistake. [5]

c) The lifetime of an article has a normal distribution with mean 400 hours and standard deviation 50 hours. Assuming normal distribution, find the expected number of articles out of 2000 whose lifetime lies between 335 hours and 465 hours. [5]

[Given : $Z = 1.3$, $A = 0.4032$]

Q6) a) If the directional derivative at $\phi = axy + byz + czx$ at $(1, 1, 1)$ has maximum magnitude 4 in a direction parallel to x -axis, find the value of a, b, c . [5]

b) Show that $\vec{F} = (2xz^3 + 6y)\vec{i} + (6x - 2yz)\vec{j} + (3x^2z^2 - y^2)\vec{k}$ is irrotational. Find scalar potential ϕ such that $\vec{F} = \nabla\phi$. [5]

c) If $\vec{F} = (2xy + 3z^2)\vec{i} + (x^2 + 4yz)\vec{j} + (2y^2 + 6xz)\vec{k}$, evaluate $\int_c \vec{F} \cdot d\vec{r}$ where c is the curve $x = t$, $y = t^2$, $z = t^3$ joining $(0, 0, 0)$ and $(1, 1, 1)$. [5]

OR

Q7) a) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [5]

b) Prove (any one). [5]

i)
$$\nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^n} \right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$$

ii)
$$\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^3} \right) = \frac{\bar{a}}{r^3} + \frac{3(\bar{a} \cdot \bar{r})\bar{r}}{r^5}$$

c) Using Green's theorem evaluate $\oint_c (x^2\bar{i} + xy\bar{j}) \cdot d\bar{r}$ where c is the boundary of region enclosed by $y = x^2$ and line $y = x$. [5]

Q8) a) If $f(z) = u + iv$ is analytic, find $f(z)$ in terms of z if given, $u = x^2 + y^2$. [5]

b) Using Cauchy's integral formula, evaluate $\oint_c \frac{z^2 + 2}{(z-1)^2} dz$, where c is the circle $|z| = 2$. [5]

c) Find the bilinear transformation which maps the points $-i, 0, 2 + i$ of the z -plane on to the points $0, -2i, 4$ of the w -plane. [5]

OR

Q9) a) If $f(z) = u + iv$ is an analytic function, find $f(z)$ in terms of z if, $u - v = (x - y)(x^2 + 4xy + y^2)$. [5]

b) Evaluate $\oint_c \cot z dz$, where ' c ' is the circle $|z| = 4$ using Cauchy's residue theorem. [5]

c) Show that under the transformation $w = z + \frac{4}{z}$ the circle $|z| = 2$ is mapped onto the straight line. [5]

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