Total No. of Questions : 8]

SEAT No. :

PB3615

[Total No. of Pages : 4

[6261]-20
S.E. (Electrical Engineering)
ENGINEERING MATHEMATICS - III
(2019 Pattern) (Semester - III) (207006)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt Q.No.1 or Q.No.2, Q.No.3 or Q.No.4, Q.No.5 or Q.No.6, Q.No.7 or Q.No.8.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicates full marks.***
4) ***Use of logarithmic tables, electronic pocket calculator and steam tables is allowed.***
5) ***Assume suitable data, if necessary.***

Q1) a) Find the fourier transform of $f(x) = \begin{cases} 1 & : |x| < a \\ 0 & : |x| \geq a \end{cases}$ Hence, find the value of $\int_0^\infty \frac{\sin x}{x} dx.$ **[6]**

b) Solve any one. **[5]**

i) Find z - transform and region of convergence of $f(k) = 3(2^k) - 4(3^k)$, $k \geq 0$.

ii) Find the inverse z transform of the following

$$\frac{10z}{z^2 + 2z - 3}; |z| < 1.$$

c) Solve the difference equation. **[6]**

$6y_{k+2} - y_{k+1} - y_k = 0, y(0) = 0, y(1) = 1.$

OR

Q2) a) Solve any one. **[5]**

i) Find z-transform and region of convergence of $f(k) = \sin(3k + 2)$, $k \geq 0$.

ii) Find the inverse z-transform of the following

$$\frac{z}{(z-2)(z-3)}; |z| > 3.$$

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b) Find the fourier transform of $f(x) = \begin{cases} 1-x^2 & : |x| < 1 \\ 0 & : |x| > 1 \end{cases}$ **[6]**

c) Find $f(x)$ satisfying the integral equation **[6]**

$$\int_0^\infty f(x).\cos(wx)\,dx = e^{-w}, w \geq 0.$$

Q3) a) If –1, 1, 2 & 4 be the moments of a distribution about 5, find the moments about mean, and coefficients of Skewness & Kurtosis. **[6]**

b) The two regression equations of the variables x and y are **[6]**

$x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$

Find

i) Mean of x,

ii) Mean of y

iii) The correlation coefficient between x and y.

c) If on an average one ship in every ten is wrecked, find the probability that out of 5 ships expected to arrive, 4 at least will arrive safely. **[6]**

OR

Q4) a) The following regression equations were obtained from a correlation table **[6]**

$y = 0.516\,x + 33.73$, $x = 0.512\,y + 32.52$

Find the value of

i) the correlation coefficient

ii) the mean of x & y

b) A manufacturer knows that the rezor blades he makes contain on an average 0.5% of defectives. He packs them in packets of 5. What is the probability that a packet picked at random will contain 3 or more faulty blades? **[6]**

c) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? **[6]**

[Given that $p(z \geq 2) = 0.0228$]

Q5) a) Find directional derivative of $\phi = x^2y + xyz + z^3$ at (1, 1, 1) along the normal to the surface $x^2y^3 = 4xy + y^2z$ at the point (1, 2, 0). **[6]**

b) Show that $\bar{F} = \left(ye^{xy}\cos z\right)\bar{i} + (xe^{xy}\cos z)\bar{j} - e^{xy}\sin z\bar{k}$ *is* irrotational. Find corresponding scalar ϕ such that $\bar{F} = \nabla\phi$. **[6]**

c) Evaluate line integral $\int_C \bar{F}.d\bar{r}$ *for* $\bar{F} = (2y+3)\bar{i} + xz\bar{j} + (yz - x)\bar{k}$ along $x = 2t^2, y = t, z = t^3$ from t = 0 to t = 1. **[5]**

OR

Q6) a) Find the constants a&b so that the surface. **[6]**

$ax^2 - byz = (a + 2)x$ will be orthogonal to the surface.

$4x^2y + z^3 = 4$ at the point (1, –1, 2).

b) With usual notations prove (any one) **[5]**

i) For a solenoidal vector field $\bar{E}$ show that curl curl curl curl $\bar{E} = \nabla^4\bar{E}$.

ii) $\nabla^2\left[\nabla.\frac{\bar{r}}{r^2}\right] = \frac{2}{r^4}$

c) Evaluate $\int_S\int\left(x^3\bar{i} + y^3\bar{j} + z^3\bar{k}\right).d\bar{S}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. **[6]**

Q7) a) Determine the analytic function F(z) = $u + iv$ if $u = 2x - 2xy$, also find Harmonic conjugate of u. **[6]**

b) Evaluate $\int_C \frac{\sin\pi z^2 + 6z}{(z-1)(z+2)}dz$ where c is the circle $|z| = 4$. **[6]**

c) Find the bilinear transformation that maps the points $z = 1, i, -1$ into the points $w = i, o, -i$ **[6]**

OR

Q8) a) Determine the analytic function $F(z) = u + iv$ if $u - v = x^3 + 3x^2y - 3xy^2 - y^3$. **[6]**

b) Evaluate $\oint_C \frac{2z^2 + z + 5}{\left(z - \frac{3}{2}\right)^2} dz$; where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. **[6]**

c) Show that under the transformation $w = \frac{i - z}{i + z}$, x-axis in z-plane is mapped onto the circle $|w| = 1$. **[6]**

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