Total No. of Questions : 9]

SEAT No. : ____

PA-1207

[Total No. of Pages : 4

[5925]-229
S.E. (Electrical)
ENGINEERING MATHEMATICS - III
(2019 Pattern) (Semester - III) (207006)

Time : 2½ Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Question No. 1 is compulsory.***
2) ***Attempt Q.1, Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Figures to the right indicate full marks.***
5) ***Use of electronic pocket calculator is allowed.***
6) ***Assume suitable data, if necessary.***

Q1) Write the correct option: **[10]**

a) Fourier sine transform of $f(x) = e^{-x}$, $x > 0$ is **[2]**

i) $\frac{3\lambda}{1+\lambda^2}$ ii) $\frac{\lambda}{1+\lambda^2}$

iii) $\frac{\lambda}{1-\lambda^2}$ iv) $\frac{2\lambda}{1+\lambda^2}$

b) Standard deviation of four numbers 9, 11, 13, 15 is **[2]**

i) 2 ii) 7

iii) 4 iv) $\sqrt{5}$

c) Second moment μ_2 about mean is **[1]**

i) Mean ii) Standard deviation

iii) Mean deviation iv) Variance

d) The value of λ so that the vector field

$\bar{F} = (2x+3y)\hat{i} + (4y-2z)\hat{j} + (3x-\lambda 6z)\hat{k}$ is solenoidal is **[2]**

i) –6 ii) 1

iii) 0 iv) –1

P.T.O.

e) If $f(z)=u+iv$ be an analytic function where $u=4xy$, $v=2y^2-2x^2$ the value of $f(z)$ in terms of z is **[1]**

i) $4z+2z^2i$ ii) $2z^2i$

iii) $-2z^2i$ iv) $4z-2z^2i$

f) If $f(z)=u+iv$ be analytic function where $u=4xy$ then its harmonic conjugate v will be **[2]**

i) $2y^2-2x^2$ ii) $2y^2+2x^2$

iii) $-2y^2-2x^2$ iv) y^2-x^2

Q2) a) Find Fourier Transform of $f(x)=e^{-|x|}$. **[4]**

b) Attempt any one: **[5]**

i) Find z-transform of $\left(\frac{2}{3}\right)^{|k|}$ for all k

ii) Find inverse z-transform of $F(z)=\frac{z^3}{(z-1)(z-2)^2}, |z|>2$

c) Solve, $f(k+2)+6f(k+1)+9f(k)=2^k$, if $f(0)=f(1)=0$. **[6]**

OR

Q3) a) Attempt any one: **[4]**

i) Find z - transform of $f(k)=k^2 4^k, k\geq 0$.

ii) Find Inverse z - transform of $F(z)=\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}, |z|<\frac{1}{5}$.

b) Find the Fourier cosine integral representation of the function, **[5]**

$$f(x)=\begin{cases} x, & 0<x<a \\ 0, & x>a \end{cases}$$

c) Solve the following integral equation, **[6]**

$$\int_0^\infty f(x)\sin\lambda x\, dx=\begin{cases} 1, & 0\leq\lambda<1 \\ 2, & 1\leq\lambda<2 \\ 0, & \lambda\geq 2 \end{cases}$$

Q4) a) The first four moments of a distribution about the value 4 are –1.5, 17, –30, 108. Find the moments about the mean and β_1 and β_2. **[5]**

b) Obtain regression line of *y* on *x* for the following data: **[5]**

x	6	2	10	4	8
y	9	11	5	8	7

c) 20% of bolts produced by a machine are defective. Determine the probability that out of 4 bolts. Chosen at random: **[5]**

i) No one is defective

ii) Exactly one is defective

OR

Q5) a) Calculate correlation coefficient for the given data: **[5]**

x	1	2	3	4	5	6
y	6	4	3	5	4	2

b) The random variable X has a Poisson distribution.If P(X = 1) = 0.01487, P(X = 2) = 0.04461. Then find P(X = 3). **[5]**

c) Suppose heights of students follows normal distribution with mean 190 cm and variance 80 cm^2. In a school of 1000 students, how many would you except to be above 200 cm tall? (Given, *z* = 1.1180, A = 0.3686).**[5]**

Q6) a) Find the directional derivative of $\phi = xy + yz + xz$ at (1, 1, 1) along line

$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{2}.$$ **[5]**

b) Show that vector field **[5]**

$\overline{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$ is irrotational. Also find corresponding scalar potential function ϕ such that $\overline{F} = \nabla\phi$.

c) Evaluate $\int_C \overline{F} \cdot d\overline{r}$ for $\overline{F} = x^2\hat{i} + 2xy\hat{j} + z\hat{k}$ along the curve C $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. **[5]**

OR

Q7) a) Find the directional derivative of $\phi = e^{2x-y-z}$ at (1, 1, 1) along the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{2}$. **[5]**

b) Show that (Any one): **[5]**

i) $\nabla \cdot \left(r \nabla \left(\frac{1}{r^3} \right) \right) = \frac{3}{r^4}$

ii) $\nabla^4 \left(r^2 \log r \right) = \frac{6}{r^2}$

c) Using Green's theorem evaluate $\oint_C \bar{F} \cdot d\bar{r}$ where **[5]**

$\bar{F} = (2x - \cos y)\hat{i} + x(4 + \sin y)\hat{j}$ and C is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1,\ z = 0$.

Q8) a) If $V = 4xy\left(x^2 - y^2\right)$ is harmonic find its harmonic conjugate and determine F(*z*) in terms of *z*. **[5]**

b) Evaluate $\oint_C \frac{3z+4}{z(2z+1)}\,dz$ where c is the circle $|z| = 1$. **[5]**

c) Find bilinear transformation which maps the points 1, i, –1 of *z*-plane onto the points 0, 1, ∞ of *w*-plane. **[5]**

OR

Q9) a) If $u = x^3 - xy^2$ find its harmonic conjugate and hence determine F(*z*) in terms in *z*. **[5]**

b) Evaluate $\oint_C \frac{z^3 - 5}{(z+1)^2(z-2)}\,dz$ where C is the contour $|z| = 3$. **[5]**

c) Find the map of straight line $x = 2y$ under the transformation $w = \frac{2z-1}{2z+1}$. **[5]**