Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5057]-2031

S.E. (Electrical and Instru.) (First Semester)

EXAMINATION, 2016

ENGINEERING MATHS

Paper III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Figures to the right indicate full marks.

(*ii*) Use of electronic pocket calculator is allowed.

(*iii*) Assume suitable data, if necessary.

(*iv*) Neat diagrams must be drawn wherever necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 4D + 3)y = x^3 e^{2x}$

(*ii*) $(1+x)^2 \dfrac{d^2 y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2\sin[\log(1+x)]$

(*iii*) $(D^2 - 1)y = \dfrac{2}{1+e^x}$ using method of variation of parameters.

(*b*) Solve using Laplace transforms : [4]

$$\frac{d^2 y}{dx^2} + y = t,$$

given $y(0) = 1$, $y'(0) = -2$.

P.T.O.

Or

2. (*a*) A circuit consists of an inductance L and condenser of capacity C in series. An e.m.f. $\Sigma \sin nt$ is applied to it at time $t = 0$, the initial charge and initial current being zero, find the current flowing in the circuit at any time t for $\frac{1}{\sqrt{LC}} \neq n$. [4]

(*b*) Solve any *one* : [4]

(*i*) Find :

$$L\left[\frac{e^{-at} - e^{-bt}}{t}\right].$$

(*ii*) Find :

$$L^{-1}\left[\frac{s+7}{s^2+2s+2}\right].$$

(*c*) Find Laplace transform of : [4]

$$L\left[\sin t \; U(t-4)\right].$$

3. (*a*) Solve the integral equation : [4]

$$\int_0^{\infty} f(x) \cos \lambda x \; dx = e^{-\lambda}$$

where $\lambda > 0$.

(*b*) Solve any *one* : [4]

(*i*) Find z-transform of $f(k) = \frac{2^k}{k}, k \geq 1$.

(*ii*) Find inverse z-transform of :

$$F(z) = \frac{1}{(z-a)^2}, \ |z| < a.$$

(*c*) If directional derivative of : [4]

$$\phi = ax^2y + by^2z + cz^2x$$

at (1, 1, 1) has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$, hence find the values of a, b, c.

Or

4. (*a*) Attempt any *one* : [4]

(*i*) $\nabla . \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

(*b*) Find the values of the constant scalars a, b, c if the vector point function :

$$\bar{V} = (x + 2y + az)i + (bx - 3y + z)j + (4x + cy + 2z)k$$

is irrotational. [4]

(*c*) Obtain $f(k)$, given that : [4]

$$f_{k+2} - 4 f_k = 0, \ k \geq 0, \ f(0) = 0, f(1) = 2.$$

5. Attempt any *two* :

(*a*) Using Green's theorem, show that the area bounded by a simple closed curve C is given by :

$$\frac{1}{2}\int (xdy - ydx).$$

Hence find the area of the ellipse $x = a\cos\theta,\ y = b\sin\theta$. [6]

(*b*) Use the divergence theorem to evaluate : [6]

$$\iint_s (y^2z^2 i + z^2x^2 j + x^2y^2 k) \,.\, \overline{dS}$$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ above the *xoy* plane.

(*c*) Verify Stokes' theorem for : [7]

$$\overline{F} = (y - z + 2)i + (yz + 4)j + xz\,k$$

over the surface $x = 0,\ y = 0,\ z = 0,\ x = 2,\ y = 2$.

Or

6. Attempt any *two* :

(*a*) Evaluate $\int_c \bar{f} \,.\, d\bar{r}$ where

$$\bar{f} = (5xy - 6x^2)i + (2y - 4x)j$$

and c is the arc of the curve in the *xoy* plane, $y = x^3$ from (1, 1) to (2, 8) [6] (*b*) Evaluate $\iint_s \bar{F}.\overline{ds}$ where

$$\bar{F} = yzi + zxj + xyk$$

and s is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. [6]

(*c*) Use Stokes' theorem to evaluate : [7]

$$\int_c (4yi + 2zj + 6yk) \, . \, d\bar{r}$$

where c is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$.

7. (*a*) If

$$v = \frac{-y}{x^2 + y^2},$$

find u such that, $u + iv$ is analytic function. [4]

(*b*) Evaluate :

$$\oint_c \frac{z+4}{z^2 + 2z + 5} dz,$$

where c is a circle $|z - 2i| = 3/2$. [5]

(*c*) Find the bilinear transformation which maps points $0, -1, \infty$ of z-plane onto $-1, -(2+i), i$ of W-plane. [4]

Or

8. (*a*) Find the condition satisfied by a, b, c and d under which,

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is harmonic function. [4]

(*b*) Evaluate :

$$\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$$

using Cauchy's theorem. [5]

(*c*) Find the image of st. line $y = x$ under the transformation : [4]

$$\mathrm{W} = \frac{z-1}{z+1}.$$