Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5459]-195

S.E. (Comp/IT) (II Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* differential equations : [8]

(*i*) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = e^{-x} x \cos x$

(*ii*) $(2x+1)^2 \dfrac{d^2y}{dx^2} + 2(2x+1)\dfrac{dy}{dx} + 4y = 4\sin\left[2\log(2x+1)\right]$

(*iii*) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{e^x}$, by using the method of variation of parameters.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x) \sin \lambda x \, dx = e^{-\lambda}, \quad \lambda > 0.$$

P.T.O.

Or

2. (*a*) An electrical circuit consisting of an inductance L, condenser of capacity C is connected in series with applied alternating emf [sin *t*] at $t = 0$. Find the current i and charge q at any time t by assuming $w^2 = \frac{1}{LC}$ and $w \neq n$. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Obtain $z[K^2]$, $K \geq 0$.

(*ii*) Obtain $z^{-1}\left[\frac{6z}{(z+2)(z-4)}\right]$, $|z| > 4$, $K \geq 0$.

(*c*) Solve the difference equation : [4]

$$x_{K+2} - 3x_{K+1} + 2x_K = 0$$

where $x_0 = 0$, $x_1 = 1$ for $K \geq 0$.

3. (*a*) The first four moments of a distribution about the value 4 are –1.5, 17, –30 and 108. Obtain the first four central moments, β_1 and β_2. [4]

(*b*) Fit a straight line of the form $Y = aX + b$ to the following data by least squares method : [4]

X	**Y**
–2	17
1	14
3	12
6	9
8	7
9	6

(c) A series of five one day matches is to be played between India and Sri Lanka. Assuming that the probability of India's win in each match as 0.6 and results of all the five matches independent of each other, find the probability that India wins the series. [4]

Or

4. (a) The height of a student in a school follows a normal distribution with mean 190 cm and variance 80 cm^2. Among the 1,000 students from the school, how many are expected to have height above 200 cm ?

[Given : Z = 1.118, A = 0.3686] [4]

(b) In a factory manufacturing razor blades, there is a small chance of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing at least one defective blade in a consignment of 10,000 packets. [4]

(c) For a bivariate data, the regression equation of Y on X is $4x + y = \mu$ and the regression equation of X on Y is $9x + y = \lambda$. Find the values of μ and λ. Also, find the correlation coefficient between X and Y, if the means of X and Y are 2 and –3 respectively. [4]

5. (a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, –2, 1) in the direction of the vector $2\bar{i} - \bar{j} - 2\bar{k}$. [4]

(b) Show that :

$$\bar{F} = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$$

is irrotational. Hence find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. [4]

(c) Find the work done in moving a particle in the force field :

$$\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$$

along the straight line from (0, 0, 0) to (2, 1, 3). [5]

Or

6. (a) Show that (any *one*) : [4]

(i) $\nabla \cdot (\phi\nabla\psi - \psi\nabla\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$

(ii) $\nabla \cdot \left[r\nabla\left(\frac{1}{r^5}\right)\right] = \frac{15}{r^6}$.

(b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2$ at (2, –1, 2) along a line equally inclined with coordinate axes. [4]

(c) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r} \quad \text{for} \quad \bar{F} = (2y+3)\bar{i} + xz\bar{j} + (yz - x)\bar{k}$$

along the $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$. [5]

7. (a) Determine the analytic function $f(z) = u + iv$ if $u = 3x^2y - y^3$. [4]

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. [4]

(c) Evaluate :

$$\int_C \frac{4 - 3z}{z(z-1)(z-2)}\, dz,$$

where C is the circle $|z| = \frac{3}{2}$. [5]

Or

8. (a) Determine the analytic function $f(z) = u + iv$ if $u = 2x - 2xy$. [4]

(b) Find the image of the circle $|z - 1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$. [4]

(c) Evaluate :

$$\int_C \frac{3z + 4}{z(2z+1)}\, dz,$$

where C is the circle $|z| = 1$. [5]