Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5252]-566

S.E. (Computer/IT Engineering) (Second Semester)

EXAMINATION, 2017

ENGINEERING MATHEMATICS III

(2015 Course)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Your answers will be valued as a whole.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^{e^x}$

(*ii*) $(D^2 + 4D + 4)y = x^{-3} \; e^{-2x}$

(*iii*) $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$

(*b*) Find the Fourier transform of : [4]

$$f(x) = 1, \quad |x| \leq 1$$
$$= 0, \quad |x| > 1$$

and evaluate $\int_0^\infty \frac{\lambda \cos \lambda x}{\lambda} d\lambda$.

Or

2. (*a*) An inductor of 0.5 henries is connected in series with a registor of 6 ohms, a capacitor of 0.02 farads, a generator having alternative voltage given by 24 sin 10 t, $t > 0$ and switch k. Set up a differential equation for this circuit and find charge at time t. [4]

(*b*) Solve any one of the following : [4]

(*i*) Find $z\{f(k)\}$, where $f(k) = 3^k, k < 0$

$= 2^k, k \geq 0$

(*ii*) Find :

$$z^{-1}\left\{\frac{z^2}{z^2+1}\right\}$$

by using inversion integral method.

(*c*) Solve the following difference equation : [4]

$y(k+2) - 5y(k+1) + 6y(k) = 36$

$y(0) = y(1) = 0.$

3. (*a*) Calculate the first four central moments from the following data and hence find β_1 and β_2 : [4]

x	0	1	2	3	4	5	6
f	5	15	17	25	19	14	5

(*b*) Fit a straight line to the following data by least square method : [4]

x	0	5	10	15	20	25
y	12	15	17	22	24	30

(*c*) The number of breakdowns of a computer in a week is a Poisson variable with $\lambda = np = 0.3$. What is the probability that the computer will operate : [4]

(*i*) with no breakdown and

(*ii*) at the most one breakdown in a week.

Or

4. (*a*) The average test marks in a particular class is 79 and standard deviation is 5. If the marks are normally distributed, how many students in a class of 200, did not receive marks between 75 and 82. Given $z = 0.8$, Area $= 0.2881$ and $z = 0.6$, Area $= 0.2257$. [4]

(*b*) An insurance agent accepts policies of 5 men of identical age and in good health. The probability that a man of this age will be alive 30 years hence is 2/3. Find the probability that in 30 years : [4]

(*i*) all five men and

(*ii*) at least one man will be alive.

(*c*) The two variables x and y have regression lines : [4]

$$3x + 2y - 26 = 0 \text{ and } 6x + y - 31 = 0$$

Find :

(*i*) the mean values of x and y and

(*ii*) correlation coefficient between x and y.

5. (*a*) Find the directional derivative of a scalar point function $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of a vector $4i + 2j + 4k$. [4]

(b) Show that the vector field :

$$\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$$

is irrotational and hence find a scalar potential function ϕ such that $\bar{F} = \nabla\phi$. [4]

(c) Find the work done by the vector field : [5]

$$\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$$

in moving a particle of unit mass from (1, 1, 1) to (2, –1, 2).

Or

6. (a) Find the directional derivative of a scalar point function $\phi = xy - z^2 + 2xz$ at (1, 0, 2) in the direction of $4i - j + 2k$. [4]

(b) Show that (any *one*) : [4]

(i) $\nabla\left(\frac{\bar{a}.\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}.\bar{r})\bar{r}}{r^{n+2}}$, where $\bar{a}$ is a constant vector.

(ii) $\nabla^2\left(\nabla.\frac{\bar{r}}{r^2}\right) = \frac{2}{r^4}$.

(c) Evaluate the integral $\int_c \bar{F}.d\bar{r}$, along the curve $x = 2t$, $y = t$, $z = 3t$ from $t = 0$ to $t = 1$, where $\bar{F} = 3x^2i + (2xz - y)j + zk$. [5]

7. (a) If :

$$u = -2xy + \frac{y}{x^2 + y^2},$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z. [4]

(*b*) Evaluate $\oint_C \frac{e^z}{(z+1)(z+2)} dz$, where c is the contour $|z + 1| = \frac{1}{2}$. [5]

(*c*) Find the Bilinear transformation which maps the point $-i$, 0, $2 + i$ of the z-plane onto the points 0, $-2i$, 4 of the w-plane. [4]

Or

8. (*a*) If :

$$u = \frac{1}{2}\log(x^2 + y^2),$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z. [4]

(*b*) Evaluate $\oint_C \frac{\sin \pi z^2 + 2z}{(z-1)(z-2)} dz$, where c is the circle $|z| = 4$. [5]

(*c*) Find the image of the circle $(x - 3)^2 + y^2 = 2$ under the transfromation $w = \frac{1}{z}$. [4]