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S.E. (Computer Engineering/IT) (II Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Your answers will be valued as a whole.

(v) Use of electronic pocket calculator is allowed.

(vi) Assume suitable data, if necessary.

1. (a) Solve any two of the following : [8]

(i) $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ (use method of variation of parameters)

(ii) $(D^2 - 4)y = e^{4x} + 2x^3$

(iii) $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 24x$.

P.T.O.

(b) Solve the following integral equation using Fourier transform : [4]

$$\int_0^{\infty} f(x) \sin \lambda x d\lambda = 1 - \lambda, \quad 0 \leq \lambda \leq 1$$
$$= 0, \quad \lambda \geq 1.$$

Or

2. (a) An electrical circuit consists of an inductance 0.1 henry, a resistance R of 20 ohms and a condenser of capacitance C of 25 microfarads. If the differential equation of electric circuit is : [4]

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0,$$

then find the charge q and current i at any time t , given that when $t = 0$, $q = 0.05$ coulombs and $i = \frac{dq}{dt} = 0$.

(b) Solve any one : [4]

(i) Find :

$$z^{-1} \left\{ \frac{1}{(z-4)(z-5)} \right\}$$

by inversion integral method.

(ii) Find z transform of :

$$f(k) = (k+1) a^k, \quad k \geq 0.$$

(c) Using z transform, solve the following difference equation : [4]

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, \quad k \geq 0$$

$$f(0) = 0.$$

3. (a) The first four moments of a distribution about the value 4 of the variable are -1.5 , 17 , -30 and 108 . Find the central moments, β_1 and β_2 . [4]
- (b) By the method of least squares, find the straight line that best fits the following data : [4]

x	y
1	14
2	27
3	40
4	55
5	68

- (c) There is a small chance of $1/1000$ for any computer produced to be defective. Determine in a sample of 2000 computers, the probability : [4]
- (i) no defective and
- (ii) 2 defectives.

Or

4. (a) Team A has a probability of $\frac{2}{3}$ of winning whenever the team plays a particular game. If team A plays 4 games, find the probability that the team wins : [4]
- (i) exactly two games and
- (ii) at least two games.

- (b) The lifetime of a certain component has a normal distribution with mean of 400 hours and standard deviation of 50 hours. Assuming a normal sample of 1000 components, determine approximately the number of components whose lifetime lies between 340 to 465 hours. Given : [4]

$$Z = 1.2 \text{ Area} = 0.3849$$

$$Z = 1.3 \text{ Area} = 0.4032.$$

- (c) Calculate the coefficient of correlation for the following data : [4]

x	y
10	18
14	12
18	24
22	6
22	30
30	36

5. (a) Find the directional derivative of a function : [4]

$$\phi = 2x^2 + 3y^2 + z^2 \quad \text{at } (2, 1, 3)$$

in the direction of $(i + j + k)$.

- (b) Show that the vector field : [4]

$$\bar{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$$

is irrotational and hence find a scalar potential function ϕ such that $\bar{F} = \nabla\phi$.

- (c) Find the work done by a force field : [5]

$$\vec{F} = x^2i + (x-y)j + (y+z)k$$

along a straight line from (0, 0, 0) to (2, 1, 2).

Or

6. (a) Find the directional derivative of : [4]

$$\phi = 4xz^3 - 3x^2y^2z \text{ at } (1, 1, 1)$$

in the direction of a vector $3i - 2j + k$.

- (b) Show that (any one) : [4]

$$(i) \quad \nabla \left(\frac{\vec{a} \cdot \vec{r}}{r^3} \right) = \frac{\vec{a}}{r^3} - \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5}$$

where \vec{a} is a constant vector.

$$(ii) \quad \nabla^4(r^4) = 120.$$

- (c) Evaluate the integral : [5]

$$\int_C \vec{F} \cdot d\vec{r}$$

along the curve $x = y = z = t$ from $t = 0$ to $t = 2$ where

$$\vec{F} = (x^2 + yz)i + (y^2 + zx)j + (z^2 + xy)k$$

7. (a) If [4]

$$u = 3x^2y - y^3,$$

find v such that $f(z) = u + iv$ is analytic.

- (b) Evaluate : [5]

$$\oint_C \frac{z+4}{(z+1)(z+2)} dz,$$

where C is the circle $|z| < 3$.

- (c) Find the bilinear transformation which maps the points $(1, i, -1)$ from the z plane into the points $(i, 0, -i)$ of the w plane. [4]

Or

8. (a) If [4]

$$u = 3x^2 - 3y^2 + 2y,$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z .

- (b) Evaluate : [5]

$$\oint_C \frac{Az^2 + z}{z^2 - 1} dz,$$

where C is the contour $|z - 1| = \frac{1}{2}$.

- (c) Find the map of straight line $y = x$ under the transformation

$$w = \frac{z - 1}{z + 1}. \quad [4]$$