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S.E. (Computer) (I Sem.) EXAMINATION, 2019

DISCRETE MATHEMATICS

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :- (i) Neat diagrams must be drawn wherever necessary.

(ii) Figures to the right indicate full marks.

(iii) Assume suitable data, if necessary.

1. (a) Prove that the set of rational numbers is countably infinite. [3]

(b) Show that for natural no. n : [3]

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1 + 2 + \dots + n)^2.$$

(c) Let

$$X = \{1, 2, \dots, n\}$$

$$R = \{(x, y) \mid x - y\}$$

is divisible by 3. Show that R is equivalence relation. Draw the diagram for R where $n = 7$ [6]

P.T.O.

Or

2. (a) In a survey of 60 people : [3]

25 read newsweek magazine

26 read time

26 read fortune

9 read both newsweek and fortune

11 read both newsweek and time

8 read both time and fortune

8 read no magazine at all.

(i) Find the no. of people who read all the three magazines.

(ii) Find the no. of people who read exactly one magazine.

(b) Let $A = \{\phi, b\}$ construct the following sets :

(i) $A - \phi$

(ii) $\{\phi\} - A$

(iii) $A \cup P(A)$

where $P(A)$ is a power set. [3]

(c) Let

$$A = \{1, 2, 3, 4, 5\}$$

Define the following relation R on A aRb if and only if $a < b$.

Find :

(i) R in roster form

(ii) Domain and range of R

(iii) Diagraph of R .

[3]

(d) Draw Hasse diagram representing the partial ordering :

$\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$.

Find *two* examples of chain and antichain. [3]

3. (a) The company has 10 members on its board of directors. In how many ways can they elect a president, a vice president, a secretary and a treasurer ? [3]

(b) Find 8th term in the expansion of $(x + y)^{13}$. [3]

(c) Can a simple graph exist with 15 vertices, each of degree five ? [3]

(d) For which values of n, m are the following graph regular : [3]

(i) K_n

(ii) S_n

(iii) $G_{n, m}$.

Or

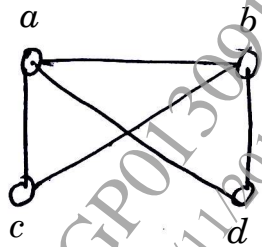
4. (a) A box contains 6 white and 5 black balls. Find number of ways 4 balls can be drawn from the box, if :

(i) Two must be white

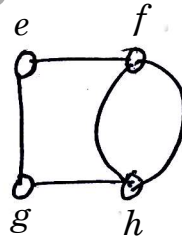
(ii) All of them must have same colour. [3]

(b) Expand $(3x - 4)^4$ using binomial theorem. [3]

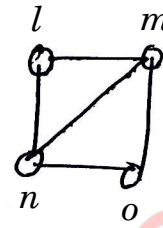
- (c) Determine whether the following graphs are isomorphic to each other. [3]



[A]



[B]



[C]

- (d) How many regions would there be in a plane graph with 10 vertices each of degree 3. [3]

5. (a) Construct a binary search tree : [4]

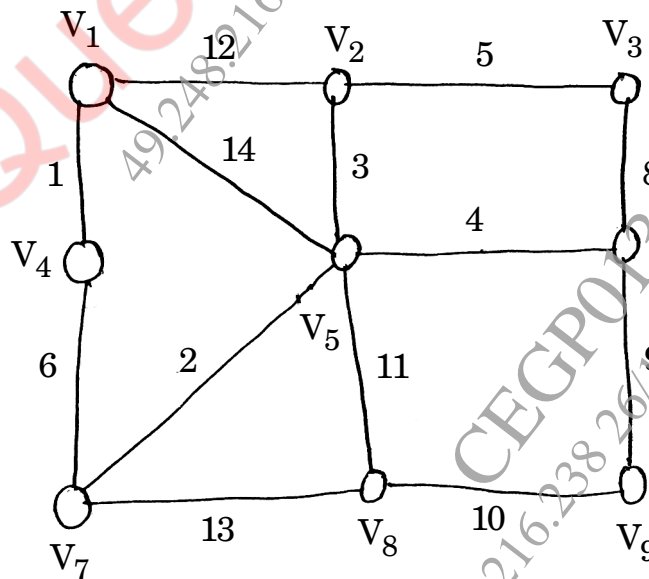
J, R, D, G, W, E, M, H, P, A, F, Q.

- (b) Construct the binary tree with prefix codes representing : [4]

(i) $a : 11, e : 0, t : 101, s : 100$

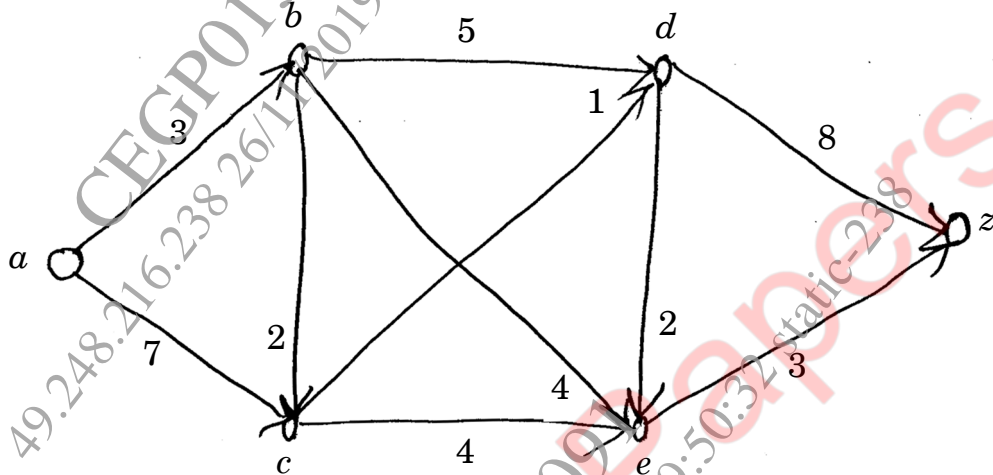
(ii) $a : 1010, e : 0, t : 11, s : 1011, n : 1001, i : 10001$.

- (c) Give the stepwise construction of minimum spanning tree using Kruskal's algorithm for the following graph. Obtain the total cost of minimum spanning tree. [5]

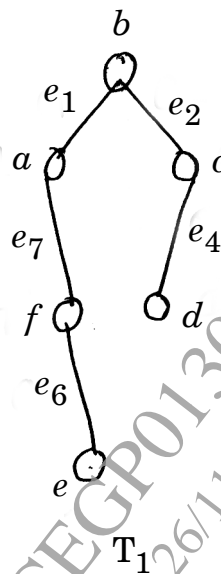
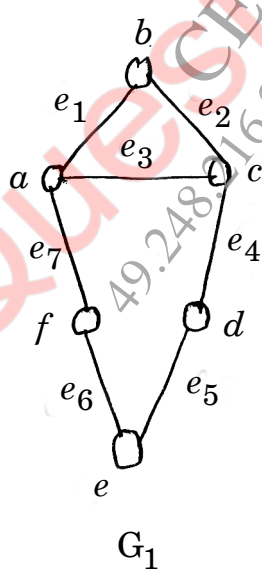


Or

6. (a) Using the labelling procedure to find maximum flow in the transport network in the following figure. Determine the corresponding minimum cut. [7]



- (b) Find fundamental cutsets and circuits of the following graph G_1 with respect to spanning tree T_1 . [4]



(c) Define with example : [2]

Level and height of a tree.

7. (a) Write properties of Binary operations. [5]

(b) Prove that the set \mathbb{Z} of all integers with binary operation $*$ defined by :

$$a * b = a + b + 1 \quad \forall a, b \in \mathbb{Z}$$

is an abelian group. [5]

(c) Let $A = \{0, 1\}$. Is A closed under : [3]

(1) Multiplication

(2) Addition.

Or

8. (a) $f : G \rightarrow G$, G is group with identity 'e' such that $f(a) \in e$ for all $a \in G$ prove that function f is homomorphism. [5]

(b) In the set \mathbb{R} of real number. Decide whether the following composition is associative $a, b, c \in \mathbb{R}$: [4]

(1) $a * b = a + 2b$

(2) $a * b = a$.

(c) Prove that the set :

$$A = \{0, 2, 4, 6, 8\}$$

with t_{10} and x_{10} operation i.e. $R = \{A, t_{10}, x_{10}\}$ is a ring. [4]