Total No. of Questions : 9]

SEAT No. :

PE4238

[Total No. of Pages : 5

[6582]-9

S.E. (Civil)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory.*
2) *Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Figures to the right indicate full marks.*
4) *Use of electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) Attempt the following :

a) If $\Sigma xy = 190, \bar{x} = 4, \bar{y} = 4, n = 10, \sigma_x = 1.732, \sigma_y = 2$ then correlation coefficient $r(x\,y)$ is **[2]**

i) 0.91287 ii) 0.8660

iii) 0.7548 iv) 0.5324

b) The mean and standard deviation of binomial probability distribution are 36 and 3 respectively. Number of trials n is **[2]**

i) 42 ii) 36

iii) 48 iv) 24

c) The curl of vector field $\bar{F} = x^2 y\,\bar{j} + xyz\,\bar{j} + z^2 y\,\bar{k}$ at point (0, 1, 2) is **[2]**

i) $4\bar{i} - 2\bar{j} + 2\bar{k}$ ii) $4\bar{i} + 2\bar{j} + 2\bar{k}$

iii) $4\bar{i} + 2\bar{k}$ iv) $2\bar{i} + 4\bar{k}$

d) ∇r^n is ________. **[2]**

i) nr^{n-1} ii) $\frac{r^{n+1}}{n+1}\bar{r}$

iii) $\frac{3r^{n-2}}{r}$ iv) $n\,r^{n-2}\bar{r}$

P.T.O.

e) $\nabla \times \bar{r}$ is [1]

i) $\bar{r}$

ii) 3

iii) $\frac{\bar{r}}{r}$

iv) $\bar{0}$

f) Line of regression y on x is [1]

i) $y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$

ii) $y + \bar{y} = r\frac{\sigma_y}{\sigma_x}(x + \bar{x})$

iii) $x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

iv) $x - \bar{x} = r\frac{\sigma_y}{\sigma_x}(y - \bar{y})$

Q2) a) The first four moments of a distribution about the value 2 are 2, 10, 20 and 25. Find first four moments about mean, coefficient of skewness and kurtosis. **[5]**

b) Calculate the coefficient of correlation for the following data. **[5]**

x	–1	1	2	4	6	7
y	–1	2	3	3	5	8

c) An unbiased coin is thrown 10 times. Find probability of getting. **[5]**

i) Exactly 6 heads.

ii) At least 6 heads.

OR

Q3) a) Obtain regression line y on x for the following data **[5]**

x	6	2	10	4	8
y	9	11	5	8	7

b) On an average 180 cars per hour pass a specified point on a particular road. Using poission distribution. Find the probability that at least two cars pass the point in any one minute. **[5]**

c) In a sample of 1000 cases, the mean score in a certain examination is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find the expected number of students scoring between 12 and 15 (both inclusive). **[5]**

(Given : $Z_1 = 0.4, A_1 = 0.1554$; $Z_2 = 0.8, A_2 = 0.2881$)

Q4) a) Find the angle between the tangent to the curve **[5]**

$\bar{r} = (t^3+2)\bar{i} + (4t-5)\bar{j} + (2t^2-6t)\bar{k}$ at $t = 0$ and $t = 1$.

b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at point (2, –1, 2) in the direction $2\bar{i} - 3\bar{j} + 6\bar{k}$. **[5]**

c) Given $\bar{u} = xyz\bar{i} + (2x^2z - y^2x)\bar{j} + xz^3\bar{k}$ **[5]**

$\phi = xy + yz + z^2$ at point (1,0,–1)

Find

i) $\nabla \cdot \bar{u}$

ii) $\nabla \times \bar{u}$

iii) $\nabla \cdot (\phi\bar{u})$

OR

Q5) a) If $\bar{r}(t) = t^2\bar{i} + t\bar{j} - 2t^3\bar{k}$ then evaluate $\int_0^1 \bar{r} \times \frac{d^2\bar{r}}{dt^2} dt$. **[5]**

b) Show that vector field $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational. Find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. **[5]**

c) For constant vector $\bar{a}$. Show that **[5]**

i) $\nabla(\bar{a} \cdot \bar{r}) = \bar{a}$

ii) $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$

Q6) a) Evaluate the line integral $\int_C \overline{F} \cdot \overline{dr}$ around the parabolic are $y^2 = x$ joining points (0, 0) and (1, 1). **[5]**

b) Evaluate $\iint_S \overline{F} \cdot \hat{n}\, ds$ where 's' is the surface of cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$ and $\overline{F} = 4xi - 2y^2 j + z^2 k$. **[5]**

c) Evaluate $\iint_S (\nabla \times \overline{F}) \cdot \hat{n}\, ds$ for $\overline{F} = yi + 3j + xk$ where 's' is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$. **[5]**

OR

Q7) a) Using Green's theorem evaluate $\int_C \cos y\, dx + x(1 - \sin y)\, dy$ where 'c' is the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, $z = 0$. **[5]**

b) Find the workdone in moving a particle once round the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, $z = 0$ under the force field given by **[5]**

$$\overline{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$$

c) Evaluate $\iint_S \overline{F} \cdot \hat{n}\, ds$ where $\overline{F} = x^3 i + y^3 j + z^3 k$ and 'S' is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. **[5]**

Q8) a) Solve $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ with boundary conditions **[8]**

i) $y(0,t) = 0$

ii) $y(l,t) = 0$

iii) $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0$

iv) $y(x,0) = k(lx - x^2)$; $0 \le x \le l$

b) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $u(x, t)$ satisfies the conditions **[7]**

i) u is finite $\forall\, t$

ii) $u(0, t) = 0$

iii) $u(l, t) = 0$

iv) $u(x, 0) = u_0$ for $0 < x < l$

OR

Q9) a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to conditions **[8]**

i) u is finite for all 't'

ii) $u(0, t) = 0$

iii) $u(l, t) = 0$

iv) $u(x,t) = \frac{3x}{l}$ for $0 \le x \le l$

b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions **[7]**

i) $u(x, \infty) = 0$, $y \to \infty$

ii) $u(0, y) = 0$

iii) $u(10, y) = 0$

iv) $u(x,0) = 100.\sin\frac{\pi x}{10}; 0 \le x \le 10$

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